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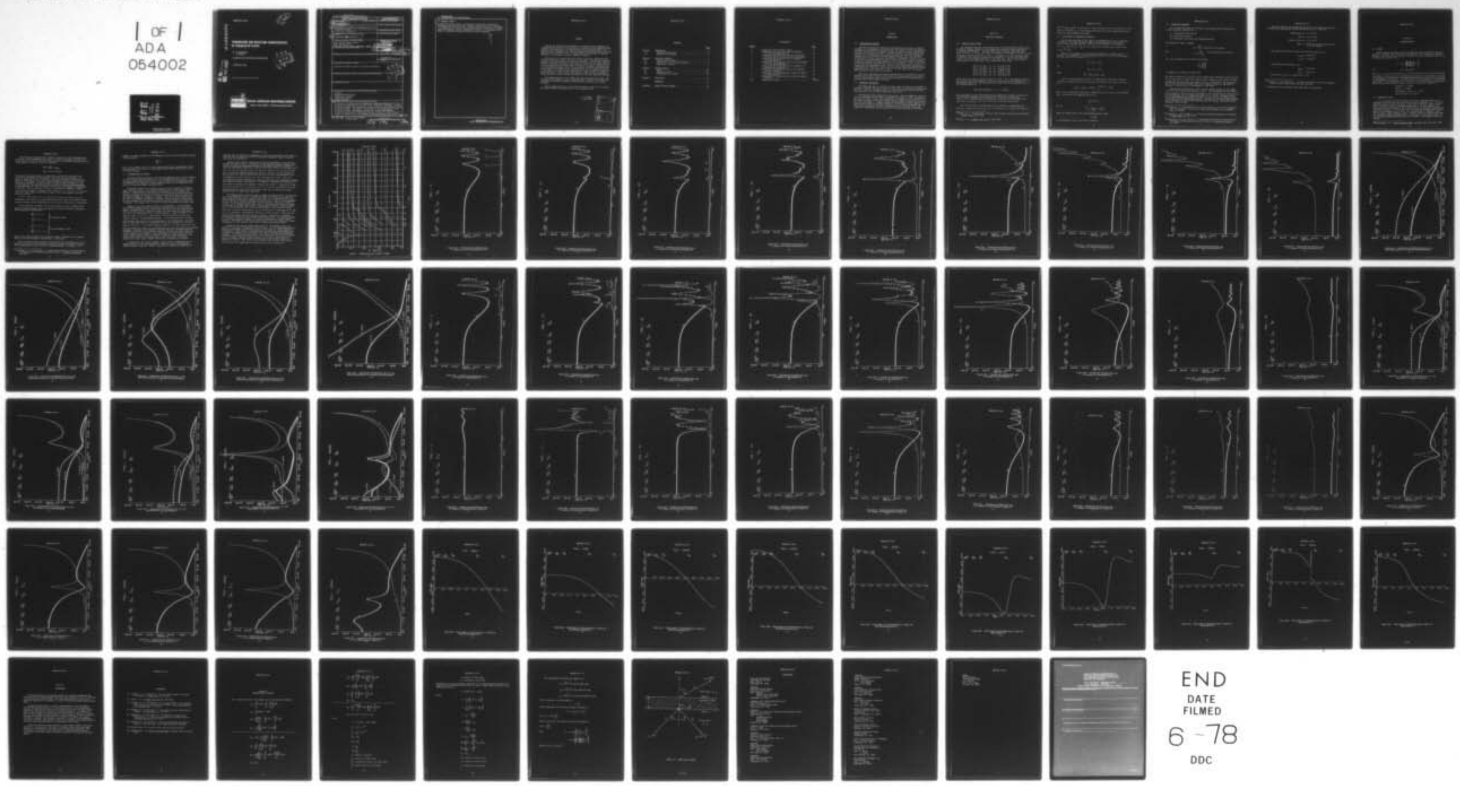

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# TRANSMISSION AND REFLECTION CHARACTERISTICS OF VISCOELASTIC PLATES


BY W. MADIGOSKY
R. FIORITO

RESEARCH AND TECHNOLOGY DEPARTMENT




1 FEBRUARY 1978



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An acoustic transmission matrix program for computing transmission (S) and reflection (R) coefficients for sound waves of arbitrary frequency, incident at arbitrary angles upon viscoelastic plates bounded by semi-infinite fluids, has been developed. The method, an extension of the formulation of Young for elastic plates, ~~(J. Young, J. Acoust. Soc. Am 59, 1500 (1976)),~~ utilizes the elastic and viscoelastic material properties of plates and the program is applied here to the problem of choosing materials for acoustic over

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20. ABSTRACT (CONT.)

windows and acoustic lenses. The materials considered are Absonic-A, polyethylene and syntactic foam. Computed results are given in the form of transmission and reflection loss and presented as functions of incidence angle and frequency, and are interpreted in terms of the propagation and material constants and modal and half wave plate resonances.

## SUMMARY

This report describes the development of a method for the computation of transmission and reflection coefficients of viscoelastic plates bounded by semi-infinite fluid media and of the phase change of the transmitted wave. The method has been implemented by a computer program which can provide the acoustic engineer with the necessary data for determining the application of specific materials to the design of acoustic windows and lenses.

The results of investigating three specific materials - Absonic-A, polyethylene and syntactic foam - are given in the report. Although the Absonic-A exhibits a greater transmission loss than polyethylene over the whole spectrum because of its higher absorption, they both have good low loss characteristics at lower frequencies and either might be chosen by a designer, based upon specific transmission windows which can be determined using the described method. A fluctuation in the phase of the transmitted wave for the Absonic-A material as the angle of incidence changes would make it a less satisfactory choice for some applications. The syntactic foam generally exhibits lower transmission losses and better defined transmission windows of the three materials analyzed.

The observed extrema in the transmission and reflection loss curves, representing such phenomena as shear absorption peaks, provide the designer with information concerning transmission characteristics heretofore not readily available.

These extrema obtained by the described method are shown to be consistent with results expected by application of the coincidence rule.

J. R. DIXON
By direction

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## ILLUSTRATIONS

# Section 1

# INTRODUCTION

## 1-1. BACKGROUND AND PURPOSE

The use of materials in underwater sound applications requires a detailed knowledge of the transmission and reflection coefficients for arbitrary angles of incidence and frequencies of propagation when the material forms a liquid/solid interface with various fluid media. Although some data, both experimental and theoretical, are available in the literature, generally they do not provide an adequate basis for the acoustic design engineer who must deal with different thicknesses, different frequencies and different fluid media. Often, as a result of variations in manufacturing processes, "off the shelf" materials may exhibit properties different from those described in the literature. Also, in some cases the relevant acoustic properties may have been inadequately characterized, and in many cases not determined at all. Finally, since experimental measurements of transmission and reflection coefficients are typically made using finite acoustic beams and plates, these experimental results are often difficult to interpret, particularly for large angles of incidence.

This report describes an analytical approach which can provide the acoustic engineer with all the necessary acoustic data (reflection/transmission coefficients and phase angle), given a set of known characteristics of the materials of interest and the interfacing fluid media.

## 1-2. MATERIALS INVESTIGATED

The method used here is valid for multiple layers of elastic or viscoelastic materials. However, the data presented in this report were generated assuming only one layer of material bounded on the input side by water and on the output side either by water or by the fluorocarbon lens fluid FC-75.

Three materials were analyzed for this report. They are Absonic-A, (an Acrylonitrile-Butadiene-Styrene plastic hereinafter referred to as ABS), polyethylene and syntactic foam. In each case the material was assumed to be in the form of a flat plate one-eighth inch thick (0.003175 meter) bounded by semi-infinite initial and final fluid media. A computer program was developed to perform computations based upon the analytical approach described in the following paragraphs with the output being produced in the form of transmission and reflection losses (dB) relative to the incident beam as functions of frequency and angle of incidence.

# Section 2

# ANALYTICAL APPROACH

## 2-1. TRANSFER MATRIX METHOD

The analytical approach used was based upon the "transfer-matrix" method first proposed by Thomson (Reference 1) and subsequently adapted by Young (Reference 2) to computer implementation. The method makes use of the fact that normal and shear stresses, $T_{zz}$ and $T_{xz}$, and normal and tangential displacements, $S_z$ and $S_x$, are continuous across a solid-solid interface. Furthermore, for each individual layer of material, the conserved stresses and displacements on one side are linear functions of the corresponding quantities on the other side. Thus a transfer-matrix representation of a solid layer can be constructed in the form:

$$\begin{bmatrix} S_x(1) \\ T_{zz}(1) \\ S_z(1) \\ T_{xz}(1) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \bullet \begin{bmatrix} S_x(2) \\ T_{zz}(2) \\ S_z(2) \\ T_{xz}(2) \end{bmatrix}$$

where (1) and (2) refer to opposite sides of a layer. For a multilayered structure of n layers, the transfer matrix for the entire structure can be constructed by multiplying togeher the individual matrices for each layer in the appropriate order. The resulting matrix is

$$[M] = [m(1)]\ [m(2)] \cdots\cdots\cdots\cdots [m(n)].$$

Using the matrix in this form and applying the appropriate boundary conditions for the fluid-solid interfaces, the reflection and transmission coefficients, R and S, can be computed, given the densities, $\rho_I$ and $\rho_F$, and the acoustic wave velocities, $C_I$ and $C_F$, of the initial and final fluid media respectively.

The $n^{th}$ solid layer is characterized by its density $\rho_n$, longitudinal and shear wave velocities, $C_{Ln}$ and $C_{Sn}$, and its thickness $d_n$. The matrix elements for

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1/ Thomson, W. T., "Transmission of Elastic Waves Through a Stratified Solid Medium," J. Appl. Phy. 21, 89 (1950)

2/ Young, J. W., J. Acoust. Soc. Am. 59, 1500 (1976)

the $n^{th}$ layer, $m_{ij}(n)$, are functions of these parameters and of the frequency, $\omega$, and angle of incidence, $\theta_I$, of an incoming plane wave and are thus computable provided values for these parameters are known.

## 2-2. APPLICATION TO VISCOELASTIC MATERIALS

For the work described in this report, the formulation of Young as described in Reference 2 has been extended to make the method applicable to viscoelastic materials by considering the wave velocities in the material to be complex; i.e., $C_{Ln} = \overset{*}{C}_{Ln}$ and $C_{Sn} = \overset{*}{C}_{Sn}$ for the longitudinal and shear waves respectively in the $n^{th}$ layer.

Figure A-1 of Appendix A illustrates this approach for a single layer of material. In the material, the wave propagation factor for both the longitudinal and shear waves must be complex, i.e.:

$$\overset{*}{k}_L = \vec{k}'_L + i\,\vec{k}''_L$$

and

$$\overset{*}{k}_S = \vec{k}'_S + i\,\vec{k}''_S$$

where

$$\overset{*}{k}_L = \omega/\overset{*}{C}_L \text{ and } \overset{*}{k}_S = \omega/\overset{*}{C}_S$$

Also, in the viscoelastic material, the amplitude of either wave will have the form of a damped plane wave traveling in the direction of $\vec{r}$ with a velocity c, i.e.,

$$e^{i\vec{k}\cdot\vec{r}} = e^{i(\vec{k}'\cdot r + i\vec{k}''\cdot\vec{r})} = e^{\frac{i\omega}{c}\hat{1}_k\cdot\vec{r}}\, e^{-\alpha\hat{1}_k\cdot\vec{r}}$$

where $\alpha$ is the appropriate absorption coefficient and $\hat{1}_k$ is a unit vector along $\overset{*}{k}$. Thus, from the real and imaginary parts of $\overset{*}{k}$,

$$\frac{\omega}{\overset{*}{C}} = \frac{\omega}{C} + i\alpha ,$$

so that

$$\overset{*}{C} = \frac{C}{1 + \frac{i\alpha C}{\omega}} = \frac{C}{1 + i\tilde{r}}$$

gives the complex form of the corresponding velocity, where

$$\tilde{r} = \frac{\alpha C}{\omega}$$

is the absorption loss for the wave in question.

## 2-3. COMPUTATION PROCEDURE

For the method described in this report, the assumed known characteristics of the viscoelastic material are:

$C_L$ = longitudinal wave velocity

$C_S$ = shear wave velocity

$\alpha_L$ = absorption coefficient for longitudinal waves.

The procedure is then to compute:

$$\tilde{r}_L = \frac{\alpha_L C_L}{\omega}\text{, absorption loss (nepers)}$$

and

$$\delta_L = \frac{2\tilde{r}_L}{1 - \tilde{r}^2_L}\text{, the longitudinal loss factor,}$$

then use the tangential loss factor relationship,

$$2\,\frac{\delta_S}{\delta_L} = \left[\frac{C_L}{C_S}\right]^2$$

to compute $\delta_S$, $\tilde{r}_S$ and $\alpha_S$ for shear waves.

The complex velocities, $\overset{*}{C}_L$ and $\overset{*}{C}_S$, and the complex wave propagation factors, $\overset{*}{k}_L$ and $\overset{*}{k}_S$, are then used to compute the transfer-matrix elements $m_{ij}$ and obtain the final transmission and reflection coefficients, S and R, respectively. The expressions for S, R and the matrix elements used in the computer program are described in Reference 2 and are given in the appendix to this report

The procedure described here relies upon the assumed validity of the tangential loss factor relationship between $\delta_L$ and $\delta_S$. This assumption has been extensively tested for various plastics and rubbers. The relationship has been verified for polypropylene over the temperature range -50°C to +100°C. Waterman, (Reference 3) and Hartmann and Jarzynski (Reference 4) have provided corroborating data on polyethylene and polymethylmethacrylate. Work done at the Naval Surface Weapons Center (Reference 5) has provided verifying data for rubbers, specifically polybutadiene, butyl and GRS.

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3/ Waterman, H. A., "Determination of the Complex Moduli of Viscoelastic Materials With the Ultrasonic Pulse Method," Part 1 and 2 Kolloid - Z.a.Z. Polymere 192, 1-16 (1963).

4/ Hartmann, B., and Jarzynski, J., "Ultrasonic Hysteresis Absorption in Polymers," J. Appl. Phys. 43, 4304 (1972)

5/ Madigosky, W. M., and Fiorito, R., "Transmission and Reflection Characteristics of Single and Multilayered Viscoelastic Plates," J. Acoust. Soc. Am. 62, S83 (1977)

The values used for the absorption coefficient for longitudinal waves were also based upon experimental testing (References 6 and 7). They are:

$$\text{Polyethylene, } \alpha_L = 2.1 \times 10^{-4} f$$

$$\text{ABS, } \alpha_L = 2.92 \times 10^{-4} f$$

$$\text{Syntactic Foam, } \alpha_L = 1.33 \times 10^{-4} f$$

where $f$ = frequency in Hertz and the units of $\alpha_L$ are dB/meter.

The respective densities of water and FC-75 were assumed to be,

$$\rho_{Water} = 1000 \text{ kg/m}^3$$

$$\rho_{FC-75} = 1762 \text{ kg/m}^3$$

The velocities were assumed to be,

$$C_{Water} = 1471 \text{ m/sec}$$

$$C_{FC-75} = 645 \text{ m/sec}$$

All data were taken at a temperature of $5^oC$.

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6/ Hartmann, B., and Jarzynski, J., "Polymer Sound Speeds and Elastic Constants," NOL TR 72-269, (1972), NSWC, White Oak, Maryland

7/ Measured Data on 3M Syntactic Foam, NSWC, White Oak, Maryland

# Section 3

# COMPUTED RESULTS

## 3-1. FORMAT

Typical computer printouts from the transmission matrix program for the three materials considered, ABS, polyethylene and syntactic foam, are shown in Figures 2 through 9. The data shown in Figures 2 through 7 give the transmission loss, TL and the reflection loss RL in dB:

$$TL = 10 \log \left[ \left( \frac{\rho_F}{\rho_I} \right) \left( \frac{C_I}{C_F} \right) |S|^2 \right]$$

and

$$RL = 10 \log |R|^2$$

where $\rho_I$, $\rho_F$, $C_I$, $C_F$ are the densities and wave velocities for the initial and final fluid media. S and R represent transmission and reflection coefficients in terms of the displacement potentials of the transmitted and reflected waves and are described in the Appendix to this report. Figures 8 and 9 show the variation in the resulting phase angle of the transmitted wave. In addition to the plotted data, each printout provides the relevant physical characteristics of the material analyzed, i.e.,

thickness, L - meters

density, $\rho$ - kg/m$^3$

longitudinal velocity, $C_L$ - m/sec

shear velocity, $C_S$ - m/sec

## 3-2. THEORETICAL BASIS

It is of interest to note the extent to which the computed results are in agreement with those predicted qualitatively by theory (Reference 8). It is well known that plates of finite thickness, under proper conditions, support various modes of vibration known as Rayleigh and Lamb waves. In their simplest form these waves may be visualized as the in-phase sums of the multiple reflections of internal plate waves. It can be shown that the equation describing these internal waves can be separated into two parts: one corresponding to "symmetric" waves, and the other to "anti-symmetric" waves. Symmetric waves are compressional since they represent symmetric displacement of the two halves of the plate relative to the mean plane dividing the plate along its thickness. Anti-symmetric waves are flexual.

[8] Brekhovskikh, L. M., Waves in Layered Media, (Academic Press, New York, 1960)

The principle of interest here is known in the theory as the coincidence rule. It states: total transmission will occur at an angle of incidence such that the phase velocity of the incident wave along the plate coincides with the phase velocity of the waves in the plate. That is, in general,

$$\text{when} \quad \frac{C}{\sin\theta} = C_{Plate}$$

$$\text{then} \quad TL \longrightarrow 0 \text{ dB}$$

If the phase velocities parallel to the plate are plotted as functions of the variable fd, the product of the frequency in Hz and the plate thickness, d, a family of dispersion curves is obtained. Each curve represents a particular symmetric or anti-symmetric wave with a definite amplitude distribution across the thickness of the plate. A typical family of curves is shown in Figure 1. The curves shown here were generated from data obtained by experimental measurements on ABS (Reference 5). For the smallest values of fd, only two modes, (Lamb modes) are present. They are shown as $s_o$ and $a_o$, corresponding to symmetric (compressional) and anti-symmetric (flexual) plate vibrations. As the product fd increases, successive higher modes become possible, again constituting symmetric and anti-symmetric families.

As $fd \rightarrow \infty$ the curves $s_o$ and $a_o$ asymptotically approach straight lines corresponding to the velocity $v_R$ of Rayleigh surface waves, while the remaining curves approach straight lines corresponding to the velocity of shear waves in the material.

As the frequency is lowered, all the curves except $s_o$ and $a_o$ asymptotically approach vertical straight lines corresponding to certain "limiting" frequencies. These frequencies are defined by:

$$\left.\begin{aligned} \frac{2fd}{C_L} &= 1, 3, 5, -- \\ \frac{fd}{C_S} &= 1, 2, 3, -- \end{aligned}\right\} \text{For symmetric waves}$$

$$\left.\begin{aligned} \frac{fd}{C_L} &= 1, 2, 3, --- \\ \frac{2fd}{C_S} &= 1, 3, 5, --- \end{aligned}\right\} \text{For anti-symmetric waves}$$

Thus all the limiting frequencies correspond to plate thicknesses of an integral number of half wave lengths of longitudinal or shear waves.

Using dispersion curves obtained as described and the coincidence rule, it is possible to find for a given frequency and plate thickness the velocity at which total transmission occurs and then the corresponding angle of incidence. For

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5/ Madigosky, W. M., and Fiorito, R., "Transmission and Reflection Characteristics of Single and Multilayered Viscoelastic Plates," J. Acoust. Soc. Am. 62, S83 (1977).

example, at normal incidence total transmission occurs at plate thicknesses satisfying the condition

$$\frac{2fd}{C_L} = n$$

when n is an integer, that is, at the limiting frequencies corresponding to thickness resonances of the plate for longitudinal waves. Shear modes will not exist at normal incidence.

## 3-3. INTERPRETATION OF RESULTS

The theory briefly described in the previous paragraph can be used to interpret the various maxima and minima which occur in the computed TL and RL curves. The TL minima (and RL maxima) should occur at the predicted total transmission points corresponding to the symmetric and anti-symmetric modes for the longitudinal and shear (for other than normal incidence) waves.

An apparent exception to the theory occurs in cases of angles of incidence where shear modes appear. The computed transmission loss curves seem to show TL maxima at frequencies where minima would be predicted by the coincidence rule. However, a detailed analysis has shown that the coincidence rule does still hold. The apparent anomaly is caused by a resonance interaction between two modes such that both a maximum and a minimum are present. This can be seen on some of the RL curves; for example, see Figure 4. However, because the factor of absorption has been included, it overrides the minimum and only the maximum in the TL curve is observed. The minima in the TL curve for longitudinal modes still remain true.

Figures 2 and 3 show computed results for polyethylene. Data for Figure 2 were obtained assuming water as the initial fluid and FC-75 as the final fluid. Figure 3 assumes water for the initial fluid and shows results for both water and FC-75 as the final fluid. Figure 2(a) shows very clearly the first three half-wave points for longitudinal waves at normal incidence. These correspond to the symmetric waves $s_1^{\ell}$ and $s_2^{\ell}$ at approximately 300 kHz and 900 kHz and the anti-symmetric wave $a_1^{\ell}$, at approximately 600 kHz. At off angles the effect of shear waves becomes more visible with increasing angle of the incident wave. The first mode for shear waves occurs at approximately 85 kHz and corresponds to the anti-symmetric wave $a_1^{s}$. It is clearly visible at angles of 10 degrees and greater. This point illustrates the anomaly in the transmission loss curve described in the previous paragraph. Generally, the transmission loss increases with increasing angle of incidence of the input wave as would be expected. However, the loss is less than 5 dB for angles up to 45 degrees. This characteristic is more clearly shown on Figure 3 which provides loss as a function of angle for fixed frequencies. The presentation of data in this format tends to be of more use to the acoustic engineer who is more often concerned with specific frequencies of interest.

A comparison of the curves in Figure 3 shows that the transmission loss is somewhat greater for the water/plate/FC-75 condition than for the comparable water/plate/water condition. However, the degree of similarity between the two

indicates that for purposes of convenience in testing, measurements could be made in water and then corrected for the FC-75 condition by applying a simple correction loss factor for water/FC-75.

Figures 4 and 5 present computed data for ABS corresponding to that for polyethylene in Figures 2 and 3. As expected the ABS generally exhibits a higher transmission loss than the polyethylene because of its larger absorption coefficient. Since the longitudinal velocities in ABS and polyethylene differ by only about 10 percent, the half wave resonance points are nearly the same for the $s_1^{\ell}$ and $a_1^{\ell}$ waves, as can be seen by comparing Figures 2(a) and 4(a). However, the shear velocity in ABS is nearly twice that of polyethylene and the first point corresponding to the anti-symmetric wave $a_1^{s}$ does not occur until approximately 150 kHz. Also, the transmission loss peaks exhibited by the ABS for shear waves are considerably stronger than comparable peaks in polyethylene. For example, a comparison of the two at an input angle of incidence of 50 degrees - Figures 2(f) and 4(f) - shows the $a_1^{s}$ peak for ABS at 150 kHz to be 25 dB greater than the $a_1^{s}$ peak for polyethylene at 85 kHz. This difference is caused by the much greater shear loss for ABS and the better impedance match for shear mode conversion.

The computed data for syntactic foam are shown in Figures 6 and 7. All data are for a water/foam/FC-75 system. It might be assumed that syntactic foam would be a nearly ideal substance for an acoustic window since it is advertised to be a "ρc" material having a near perfect impedance match with water. However, this phenomenon is true only for an infinite half plane of water/syntactic foam, and at normal incidence. As the computed results show, there is considerable interaction at angles of incidence of the input wave between 10 degrees and 60 degrees. These resulting peaks in the transmission loss curves are due primarily to the effects of shear waves, and the points corresponding to $a_n^{s}$ at odd multiples of approximately 185 kHz, and $S_n^{s}$ at multiples of approximately 370 kHz are clearly visible. There are, however, specific windows and since syntactic foam has a relatively low absorption coefficient the peaks are sharp with good nulls between them. By a judicious choice of the frequency thickness product, the syntactic foam may be a superior window material. For example, at 300 kHz the plates analyzed for this report show a lower transmission loss for syntactic foam than for either polyehtylene or ABS.

Figures 8 and 9 show the computed phase angle of the transmitted wave as a function of the angle of incidence of the input wave. The two figures show data for plates of polyethylene and ABS respectively, in both cases in a water/plate/FC-75 configuration. Phase change in the polyethylene is smooth and slowly varying with the angle of incidence at all frequencies, the range of variation increasing with frequency. For the ABS the phase change is more complex and exhibits rapid changes in direction when a particular mode is excited in the plate. This shows up at angles of incidence between 50 and 60 degrees, and is particularly strong at a frequency of 150 kHz. This effect can be correlated with the data shown on Figures 4(f) and 5(d) illustrating the strong effect of the $a_1^{s}$ shear wave.

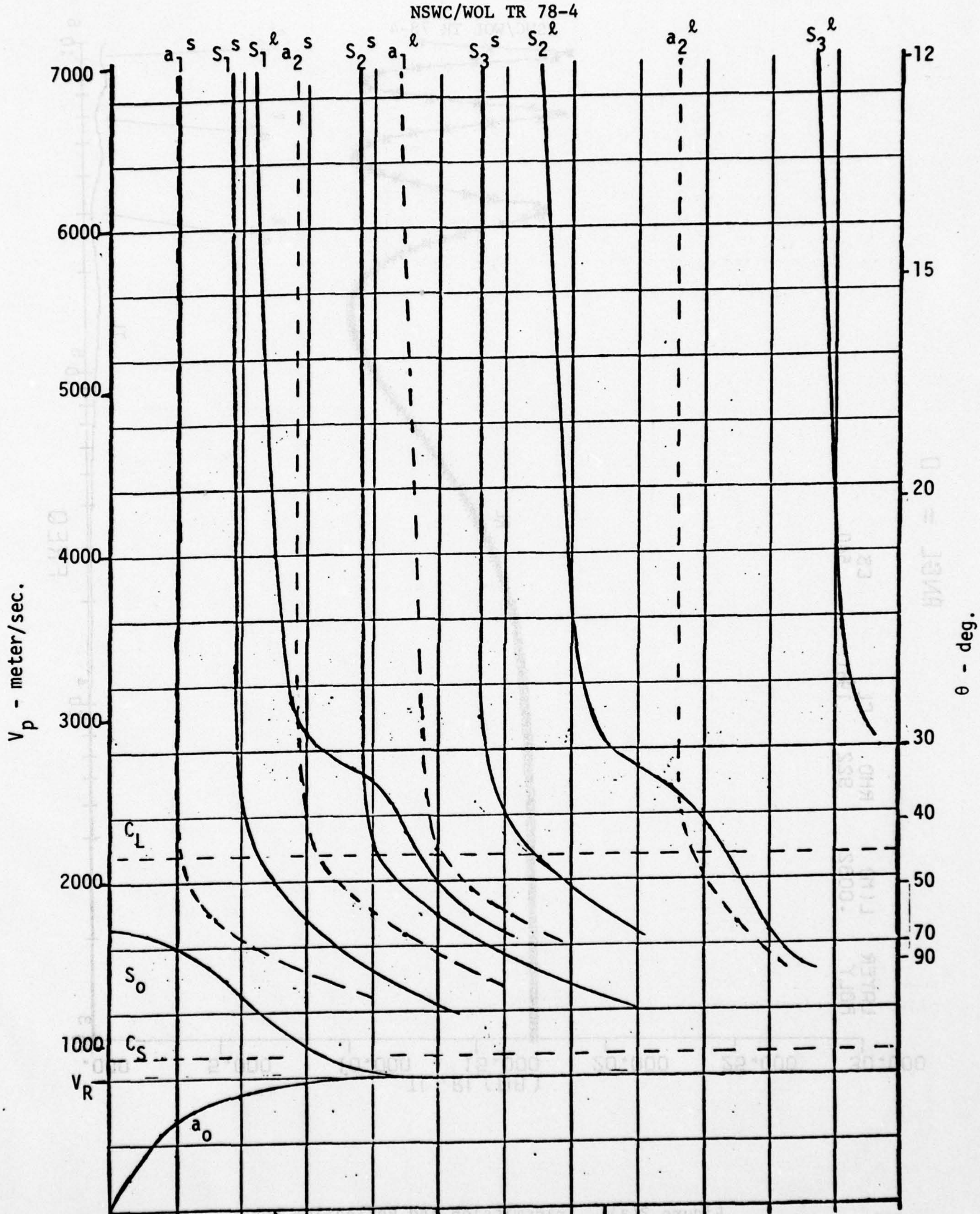


Figure 1. Dispersion Curves for ABS in Water

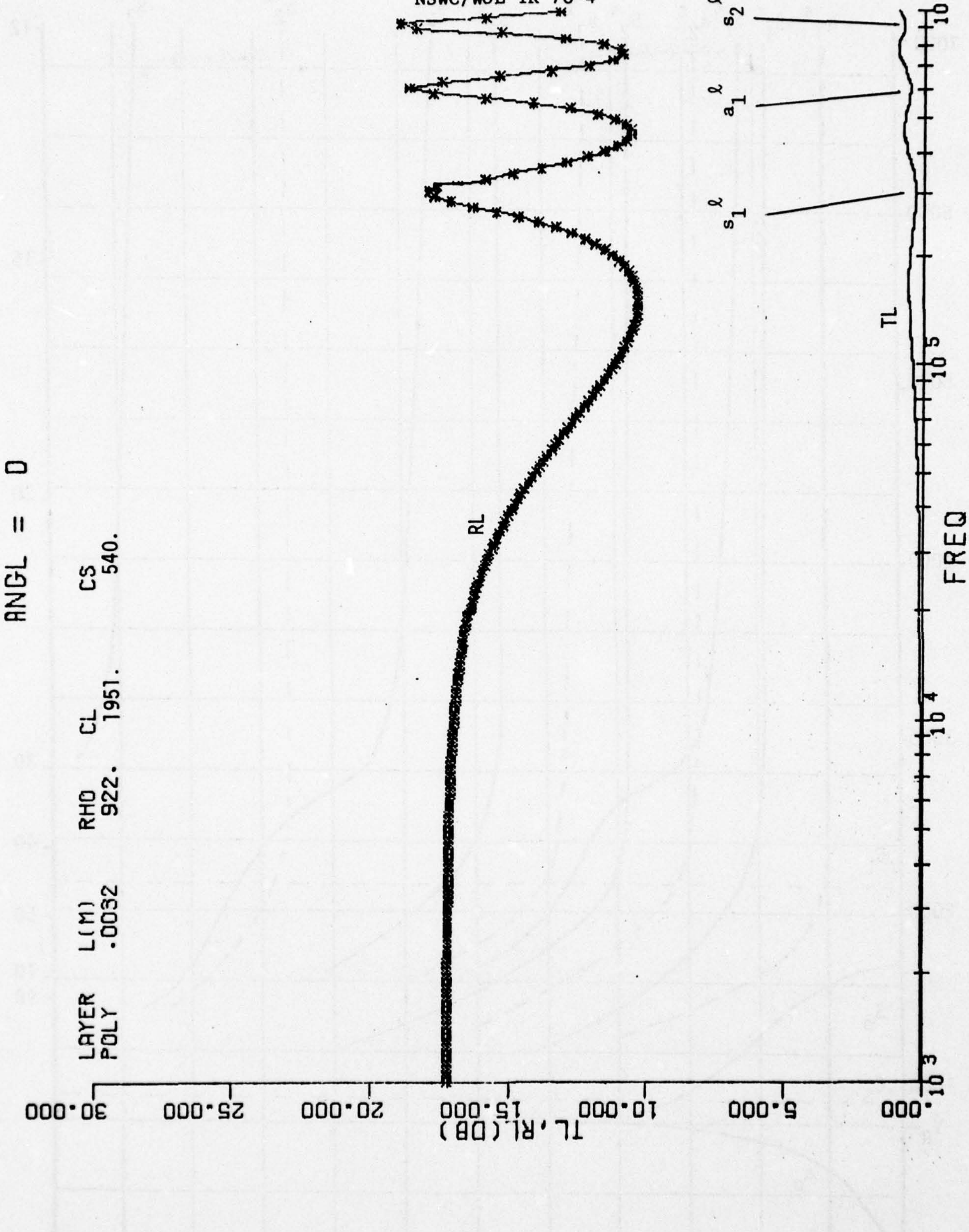


Figure 2(a). Transmission and Reflection Loss vs Frequency for Water/Polyethylene/FC-75

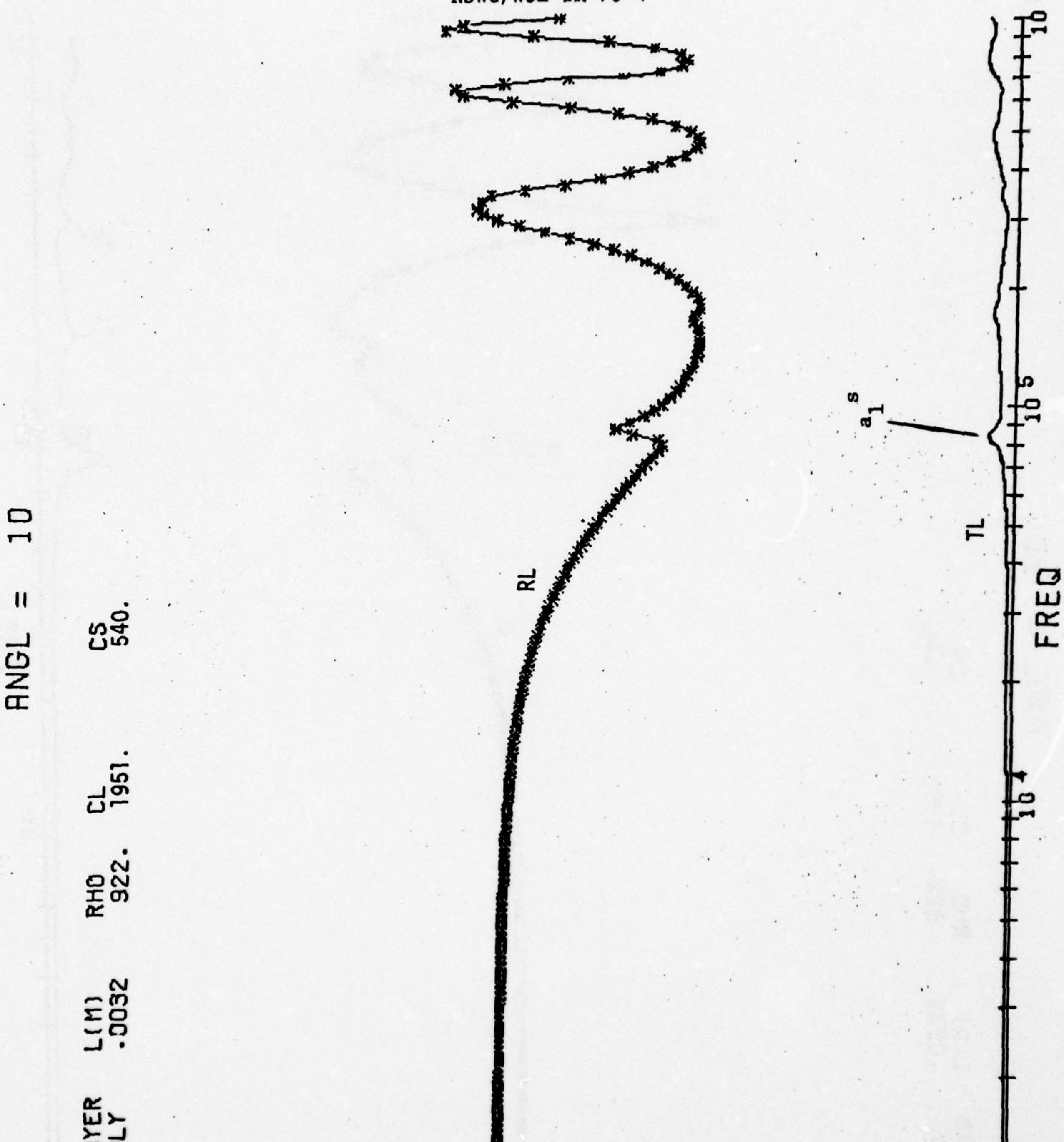


Figure 2(b). Transmission and Reflection Loss vs Frequency for Water/Polyethylene/FC-75

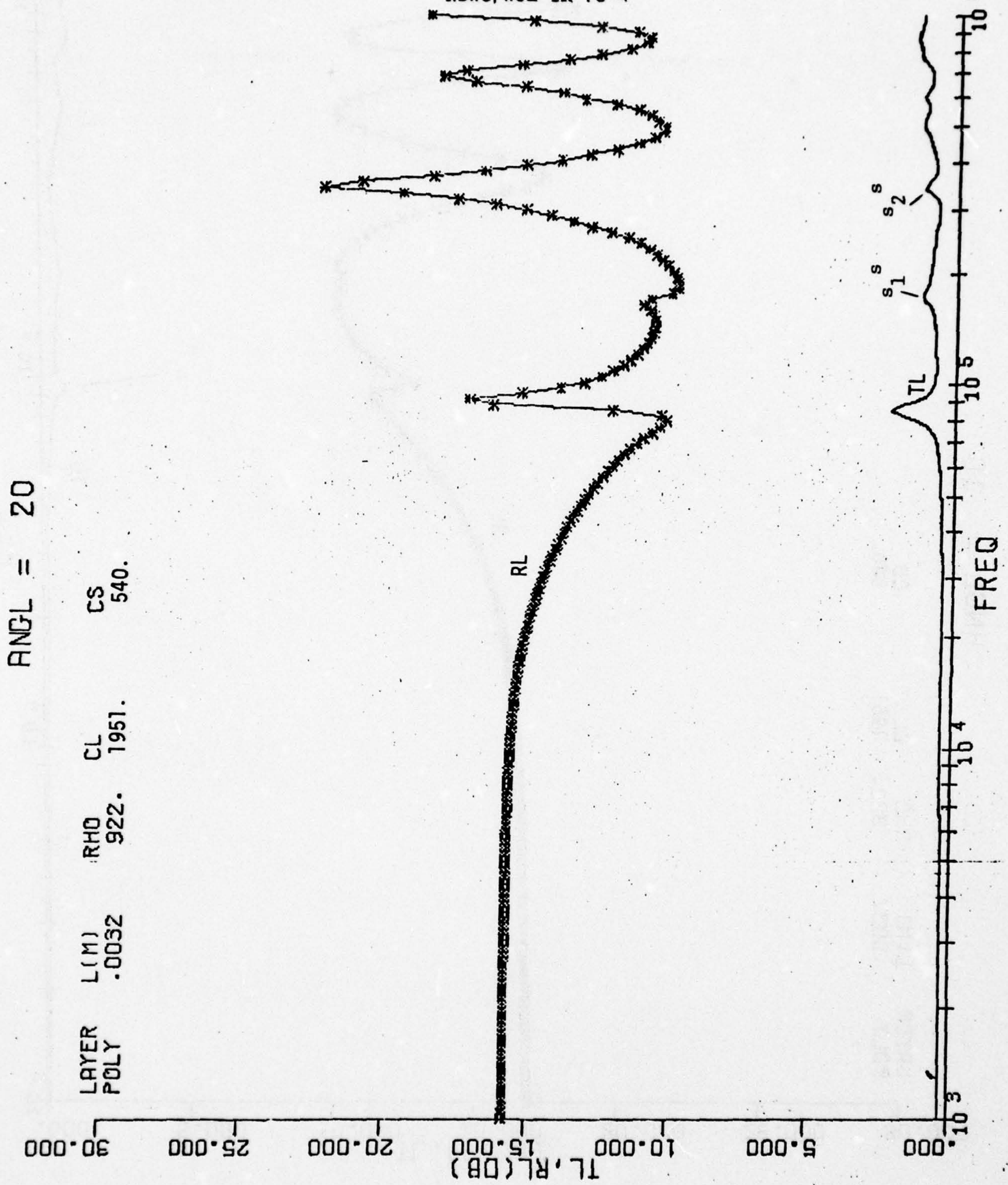


Figure 2(c). Transmission and Reflection Loss
vs Frequency for Water/Polyethylene/FC-75

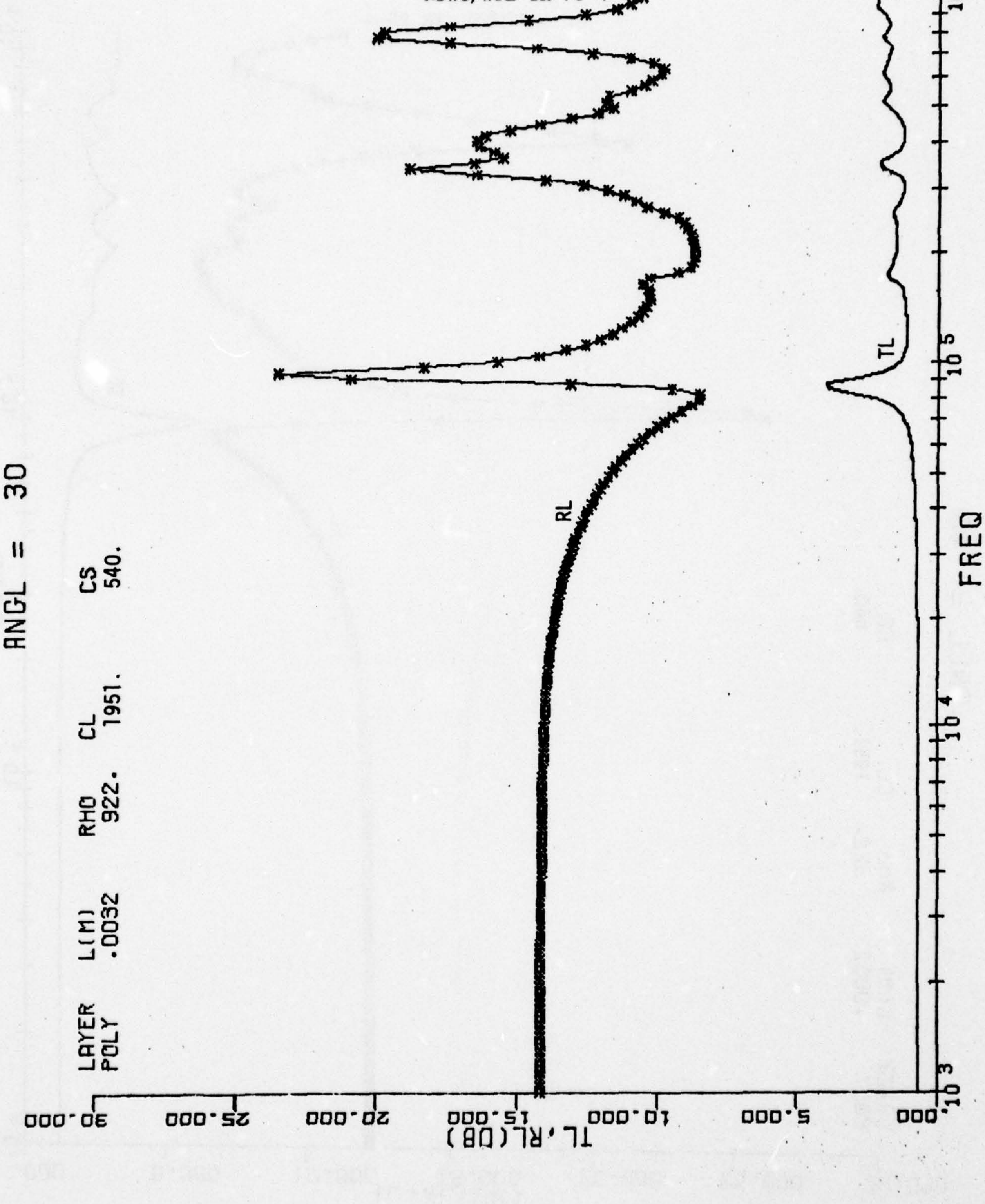


Figure 2(d). Transmission and Reflection Loss vs Frequency for Water/Polyethylene/FC-75

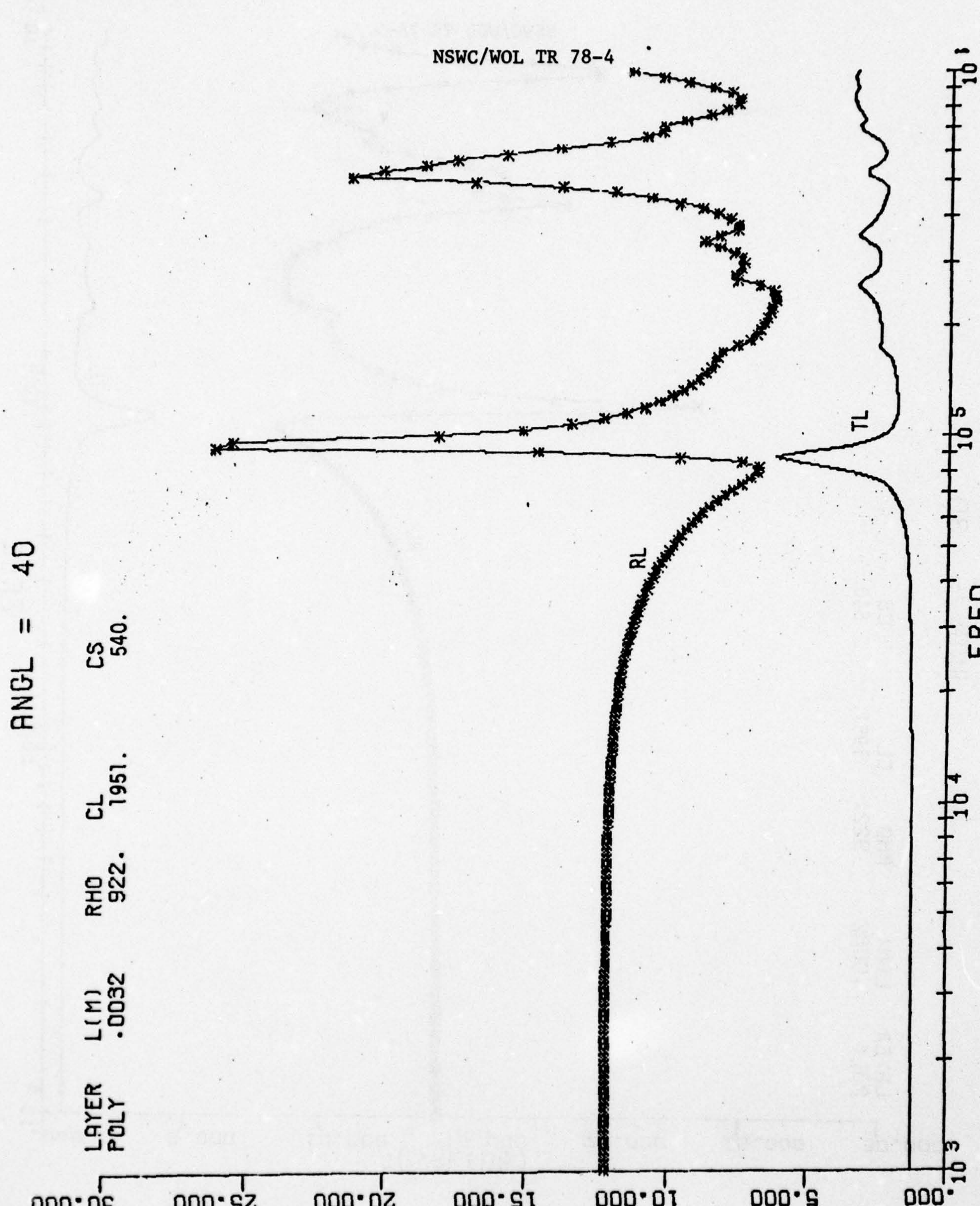


Figure 2(e). Transmission and Reflection Loss vs Frequency for Water/Polyethylene/FC-75

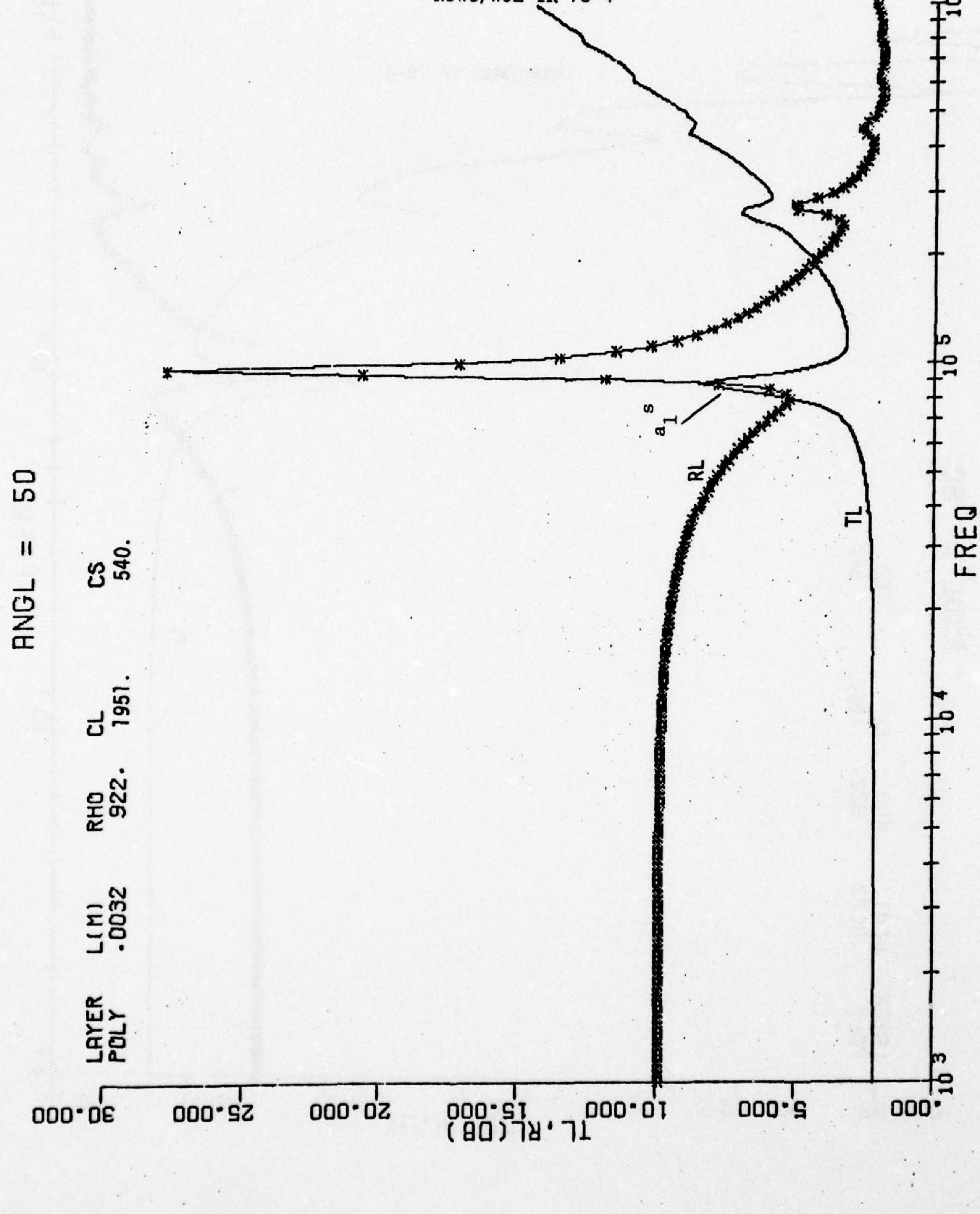


Figure 2(f). Transmission and Reflection Loss vs Frequency for Water/Polyethylene/FC-75

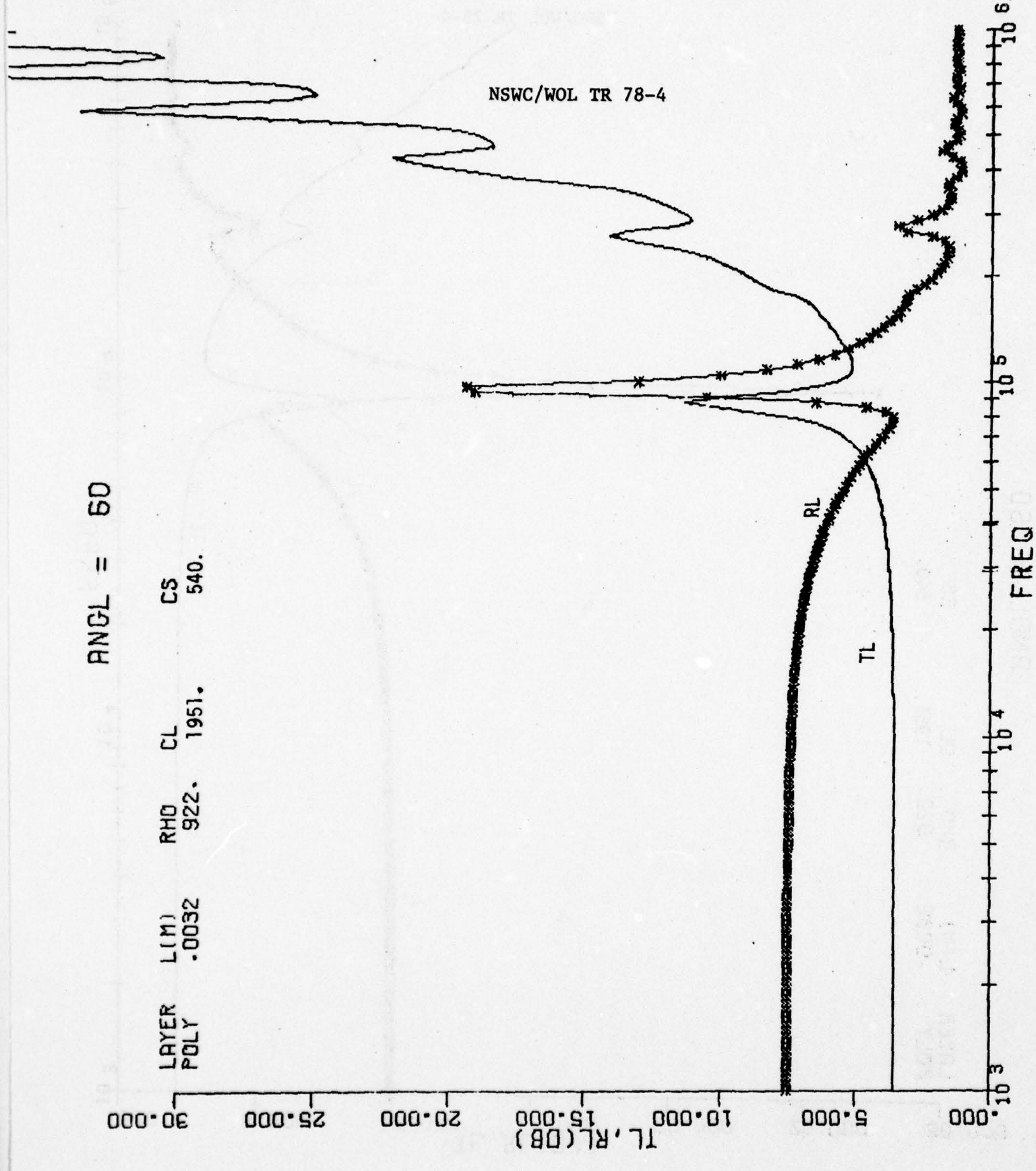


Figure 2(g). Transmission and Reflection Loss vs Frequency for Water/Polyethylene/FC-75

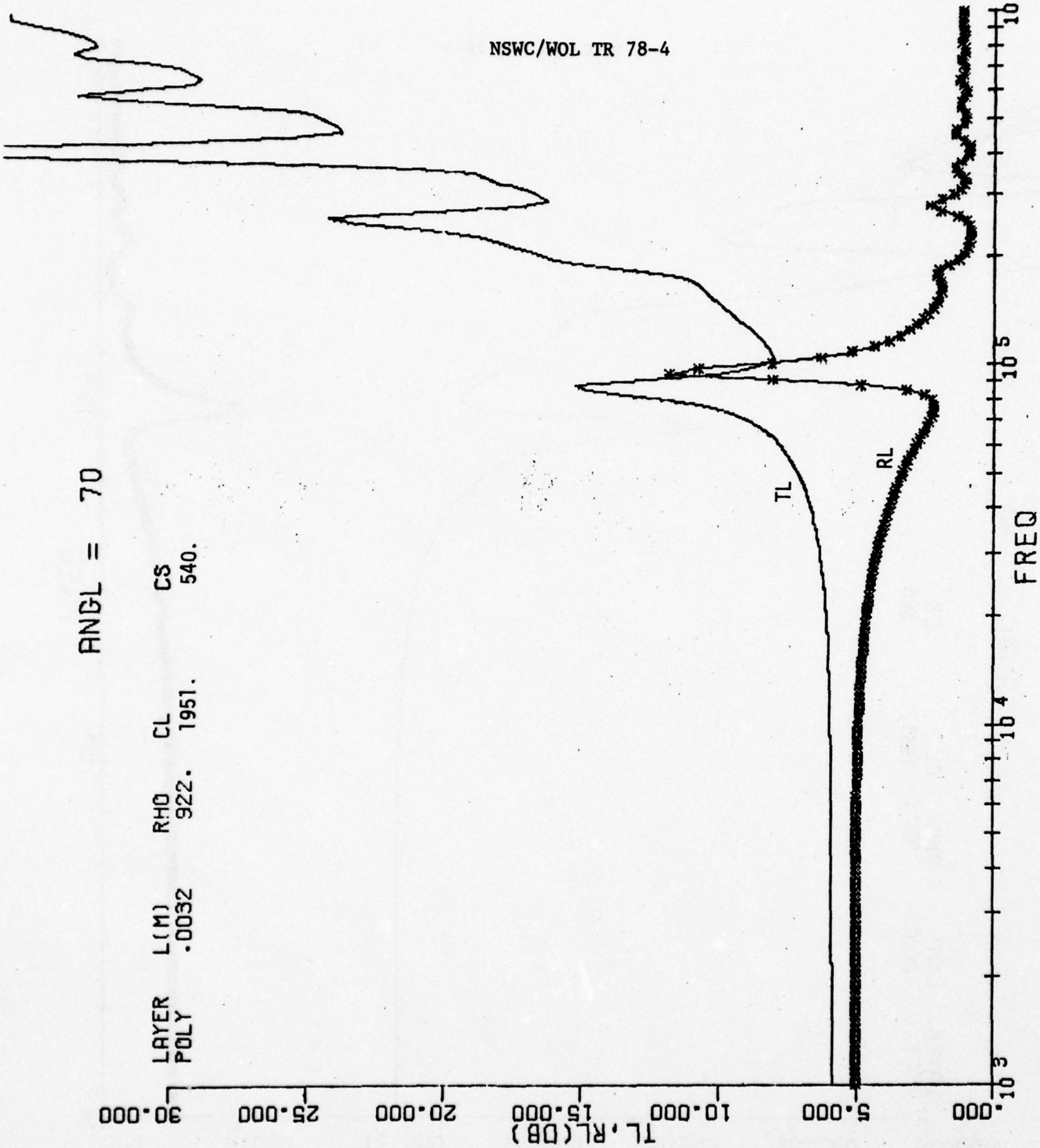


Figure 2(h). Transmission and Reflection Loss vs Frequency for Water/Polyethylene/FC-75

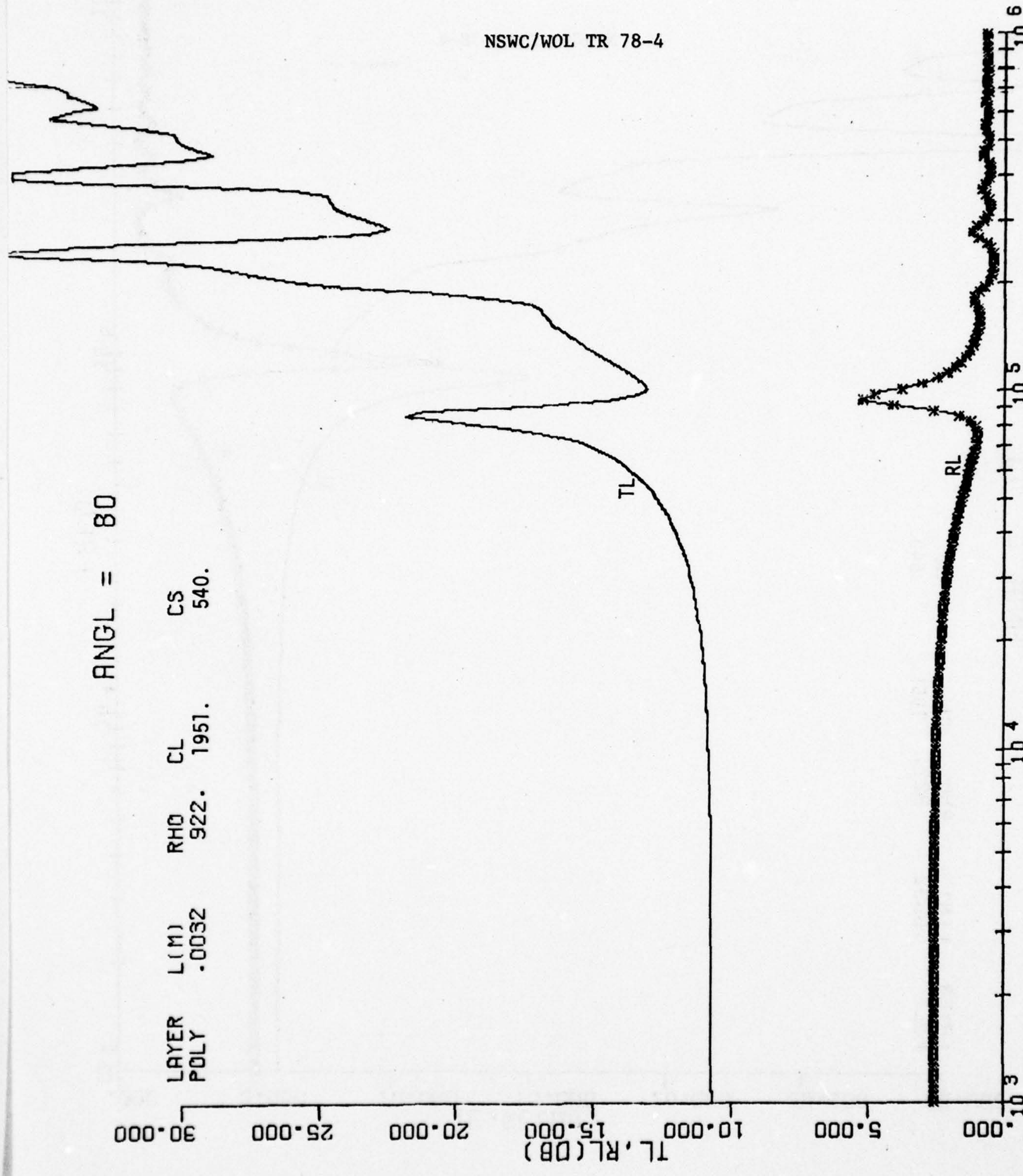


Figure 2(i). Transmission and Reflection Loss vs Frequency for Water/Polyethylene/FC-75

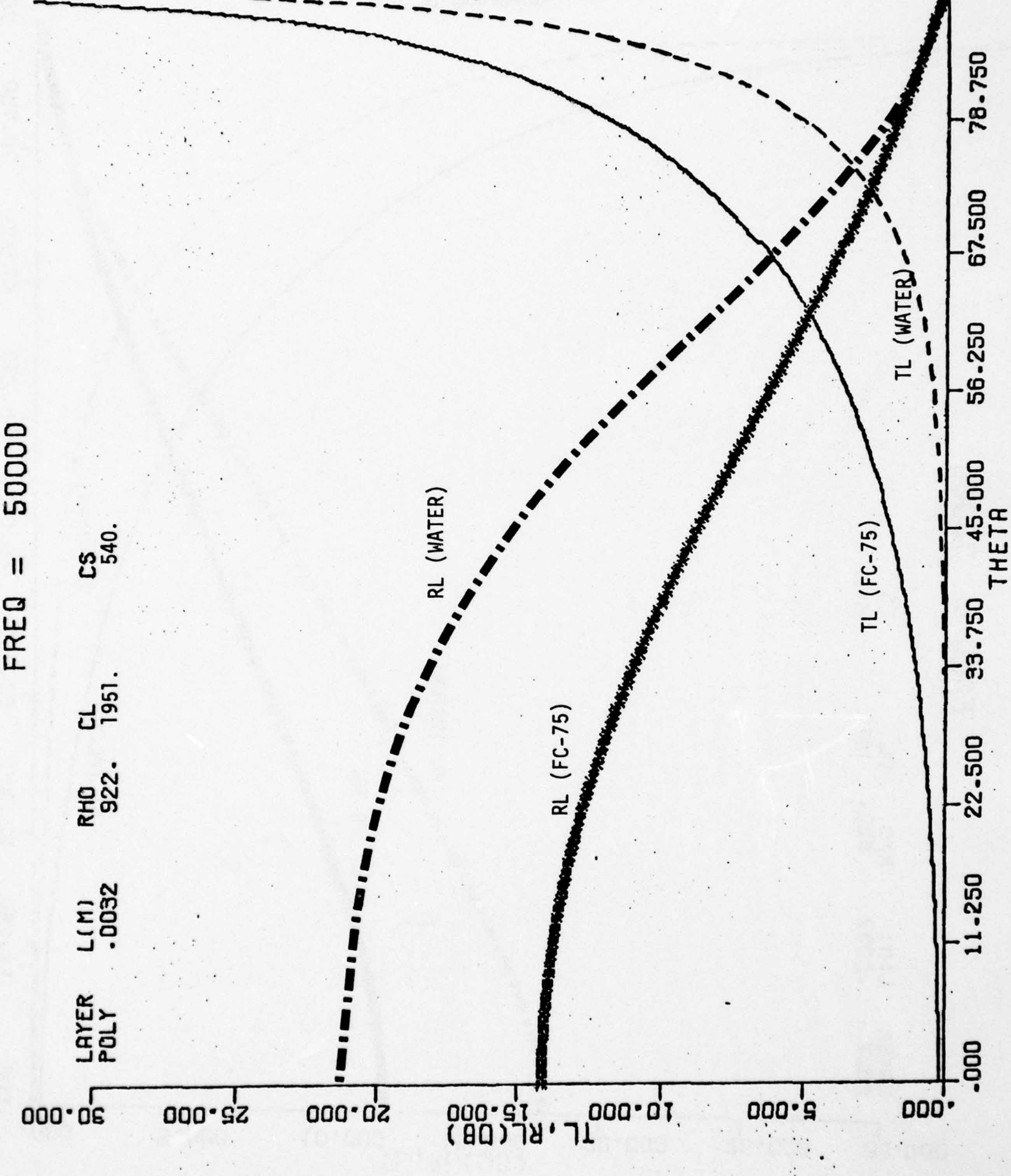


Figure 3(a). Transmission and Reflection Loss vs θ for Water/Polyethylene/FC-75 and Water/Polyethylene/Water

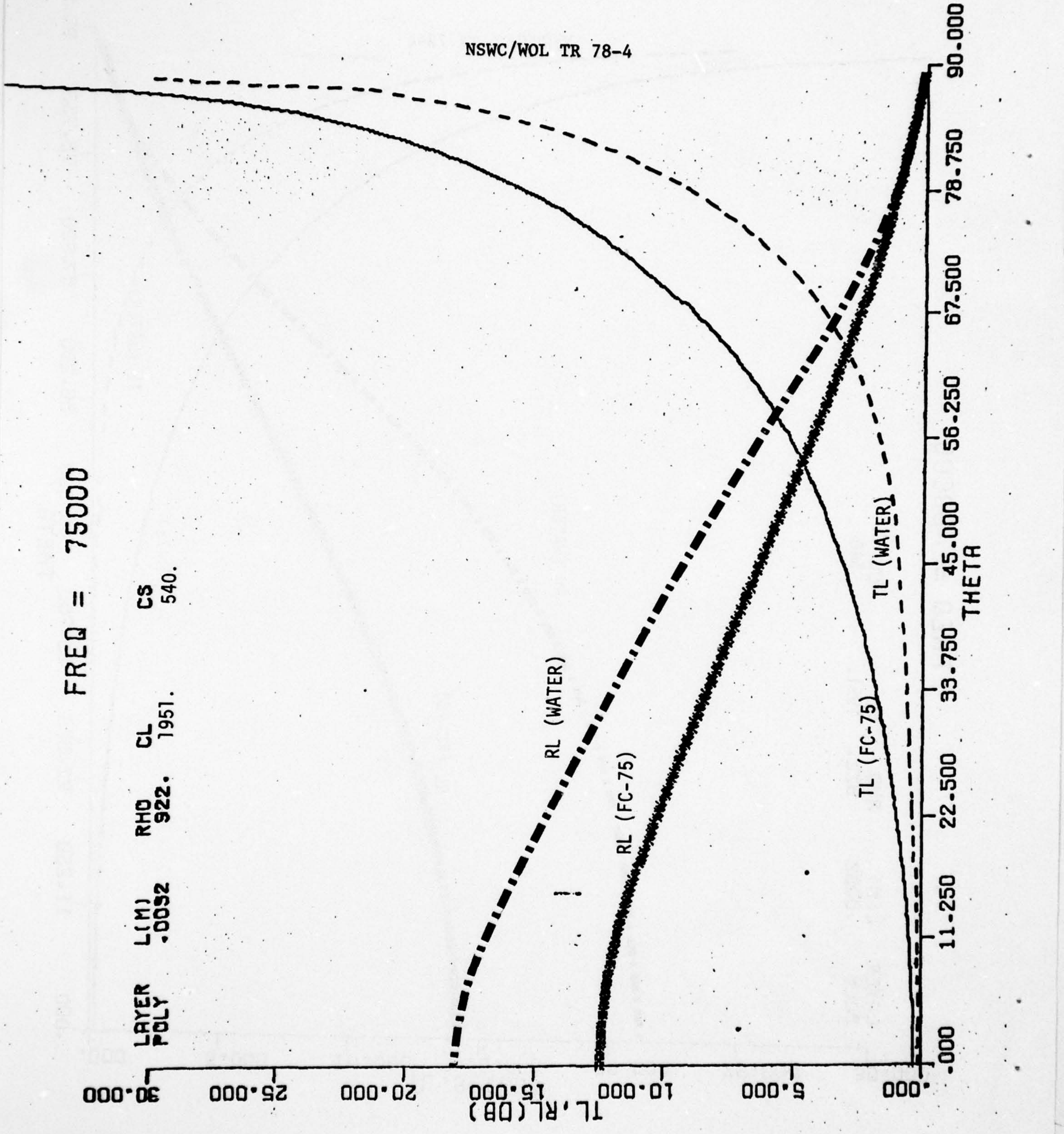


Figure 3(b). Transmission and Reflection Loss vs θ for Water/Polyethylene/FC-75 and Water/Polyethylene/Water

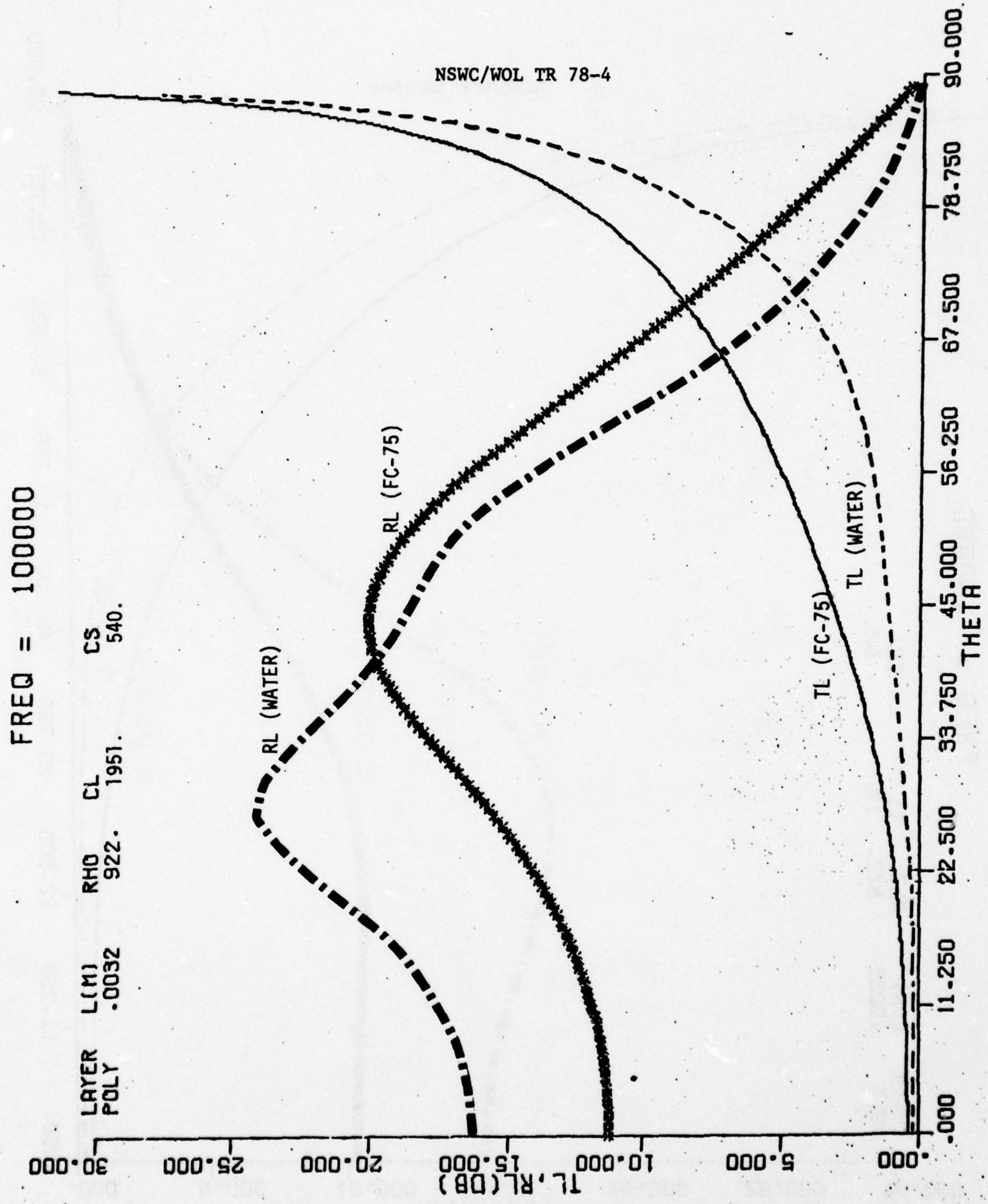


Figure 3(c). Transmission and Reflection Loss vs θ for Water/Polyethylene/FC-75 and Water/Polyethylene/Water

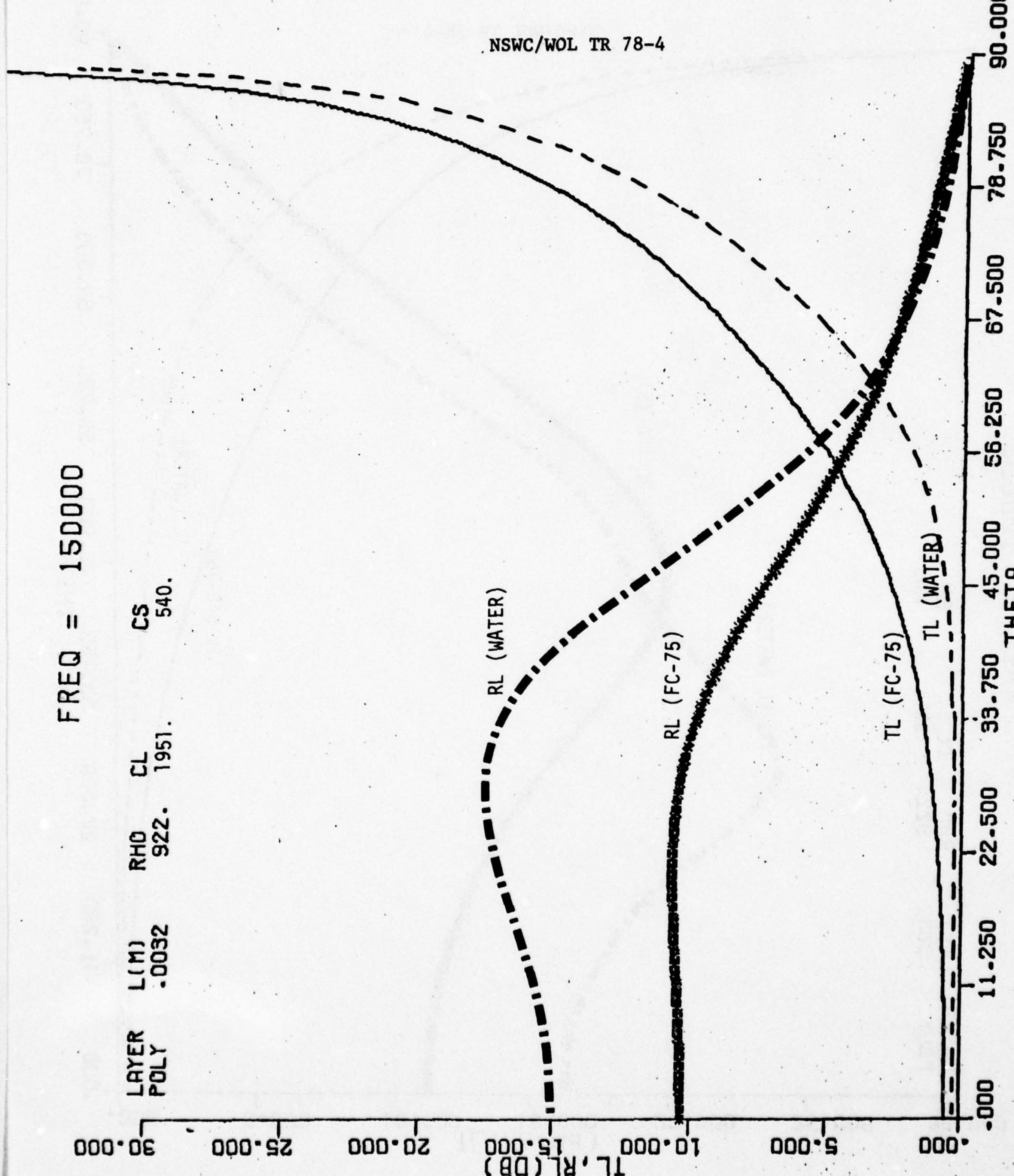


Figure 3(d). Transmission and Reflection Loss vs θ for Water/Polyethylene/FC-75 and Water/Polyethylene/Water

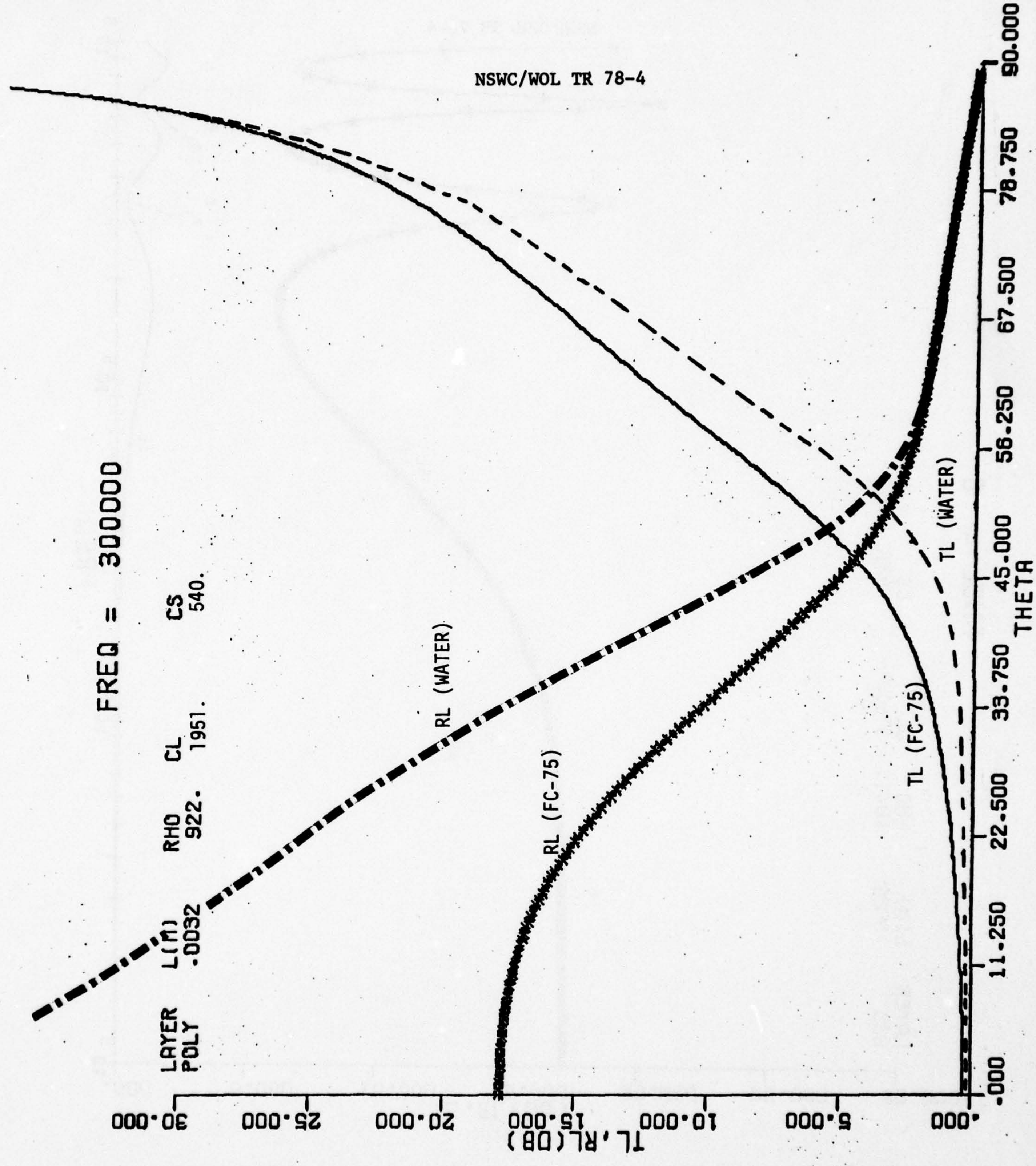


Figure 3(e). Transmission and Reflection Loss vs θ for Water/Polyethylene/FC-75 and Water/Polyethylene/Water

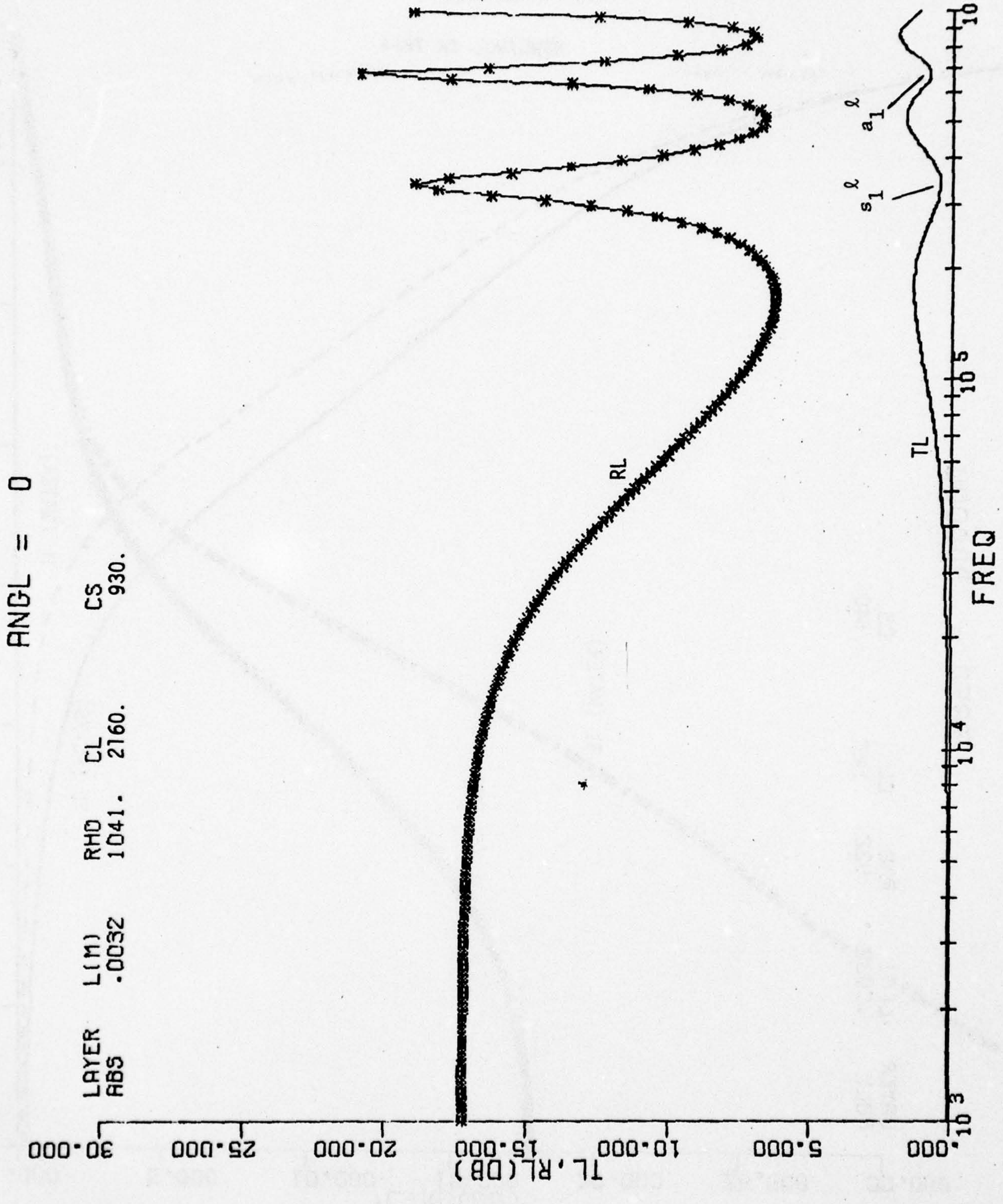


Figure 4(a). Transmission and Reflection Loss vs Frequency for Water/ABS/FC-75

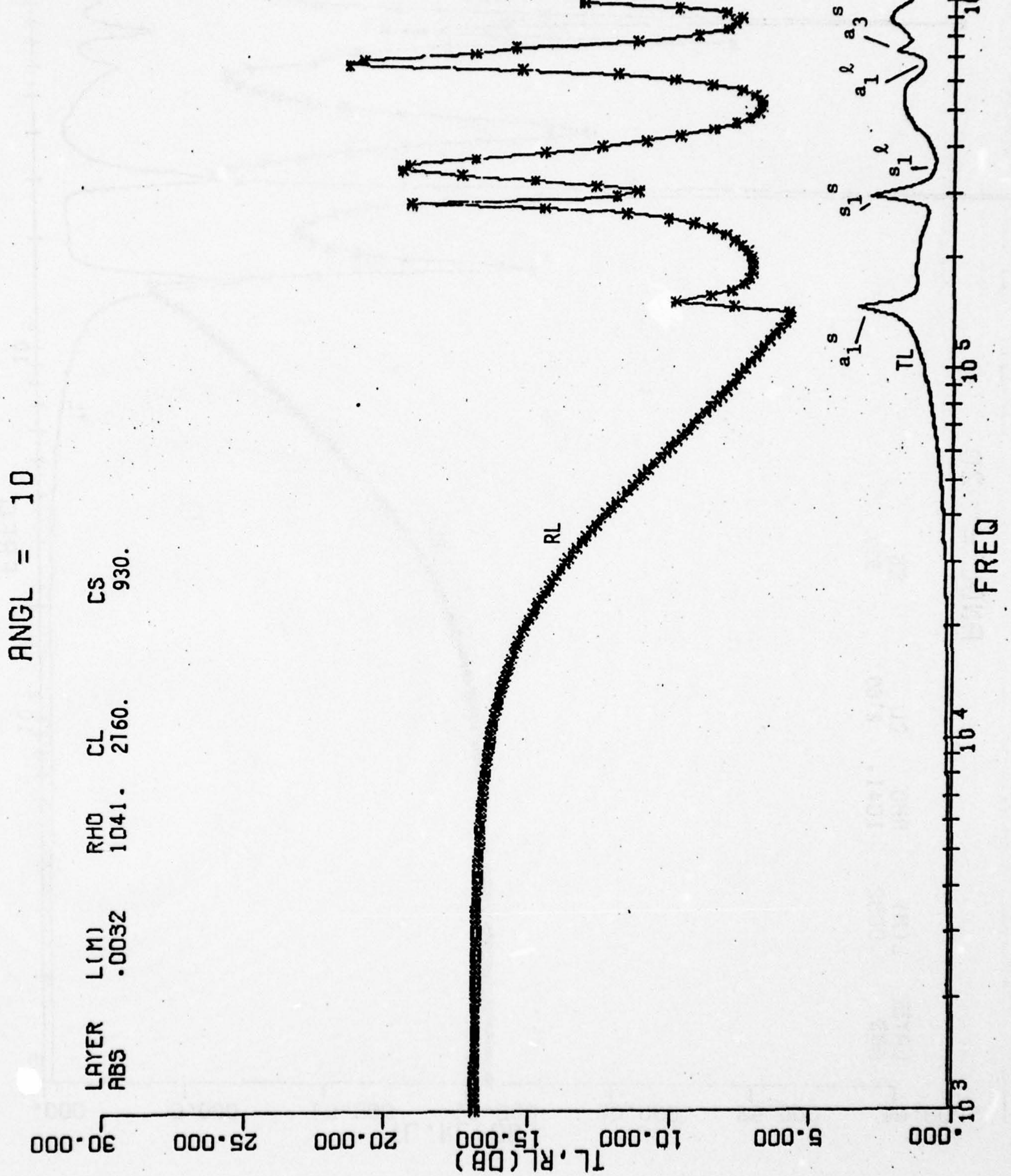


Figure 4(b). Transmission and Reflection Loss vs Frequency for Water/ABS/FC-75

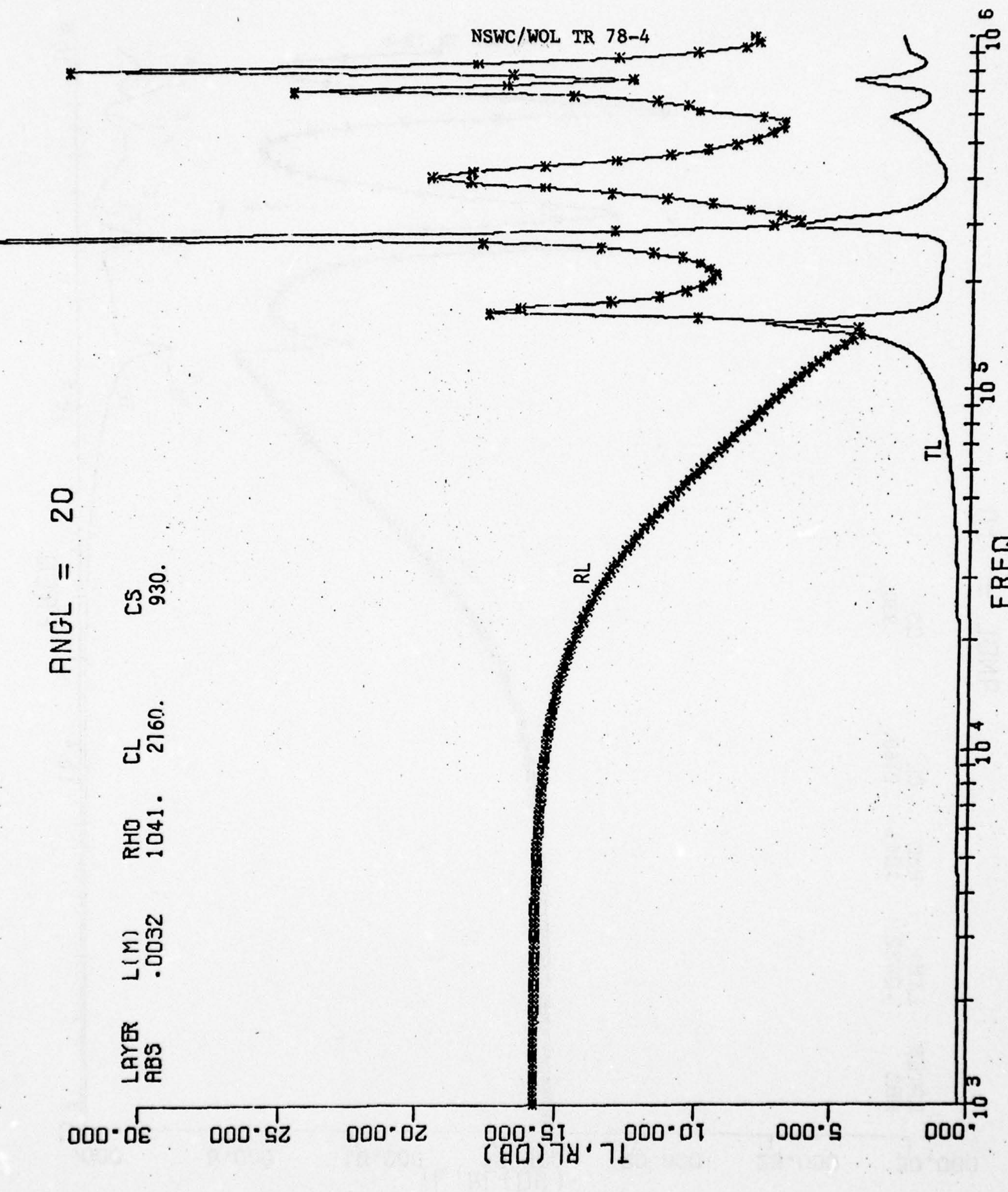


Figure 4(c). Transmission and Reflection Loss vs Frequency for Water/ABS/FC-75

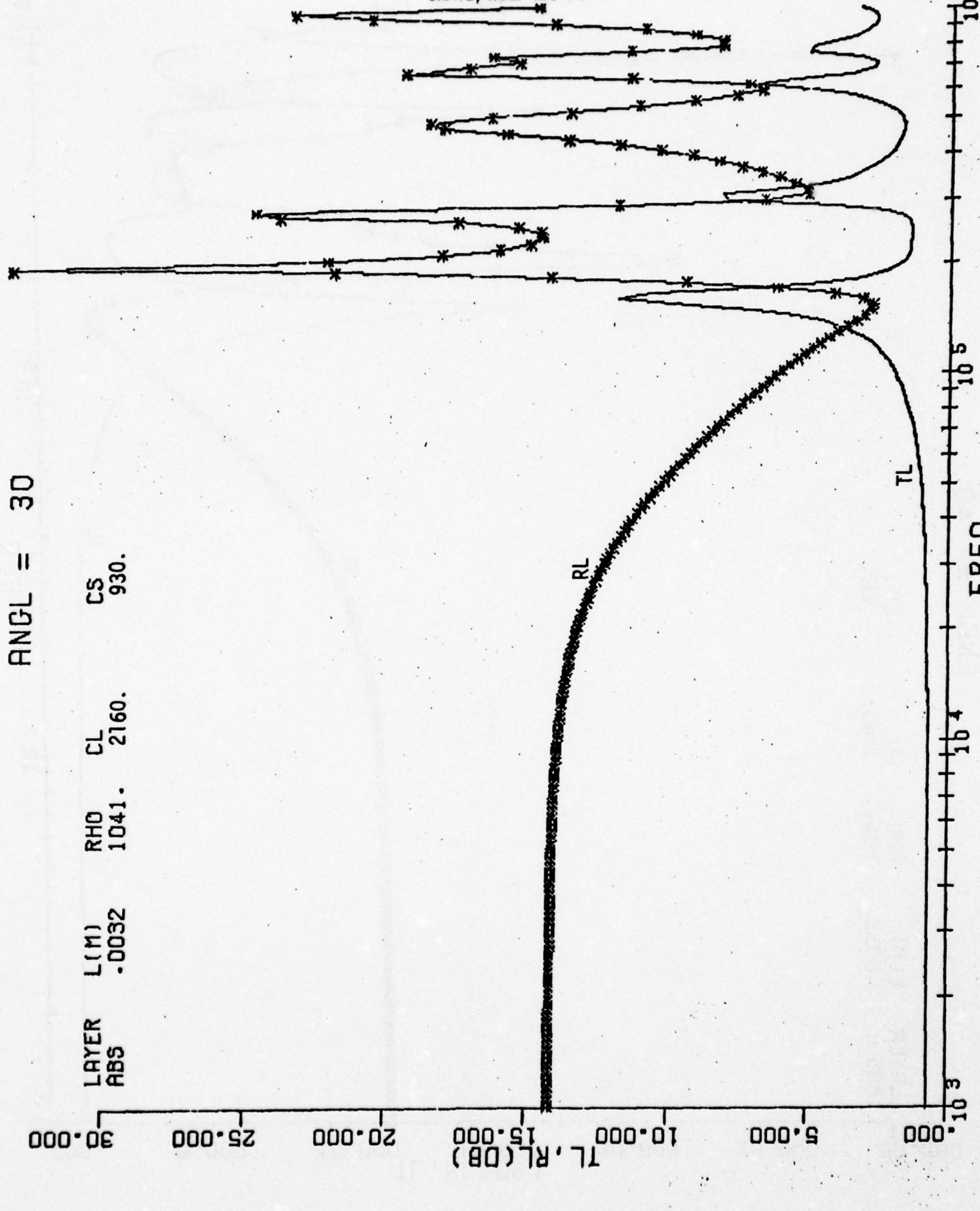


Figure 4(d). Transmission and Reflection Loss
vs Frequency for Water/ABS/FC-75

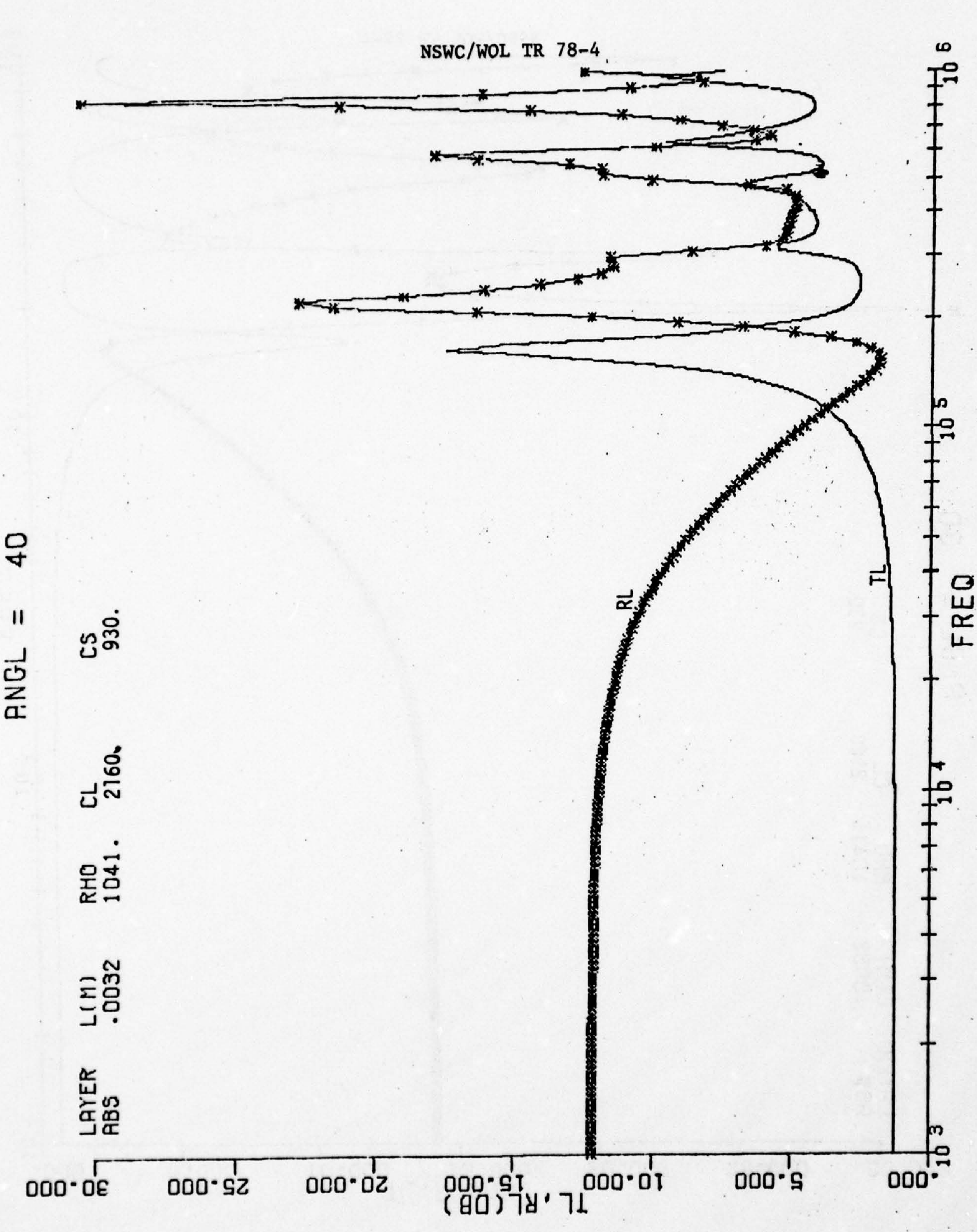


Figure 4(e). Transmission and Reflection Loss vs Frequency for Water/ABS/FC-75

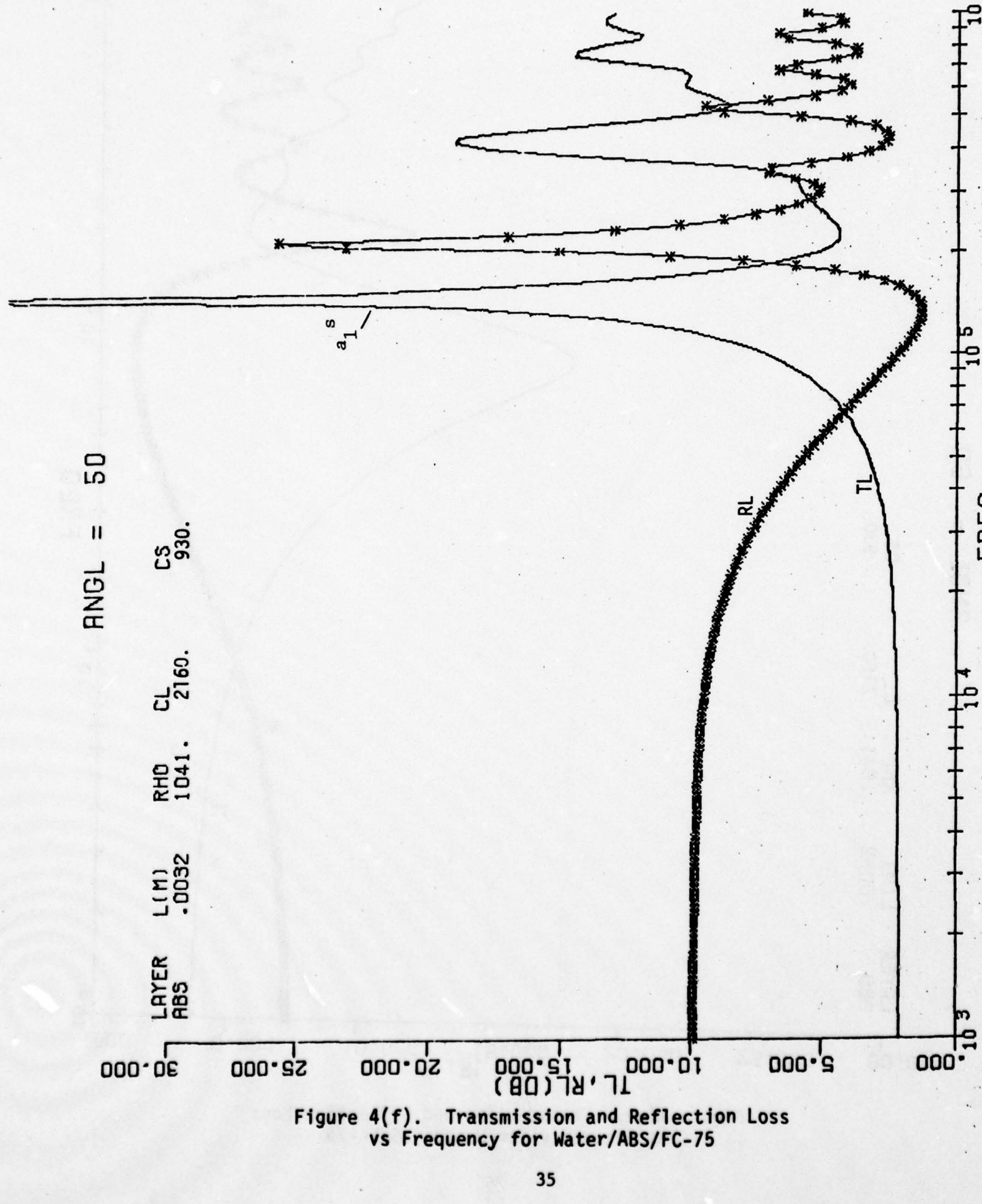


Figure 4(f). Transmission and Reflection Loss vs Frequency for Water/ABS/FC-75

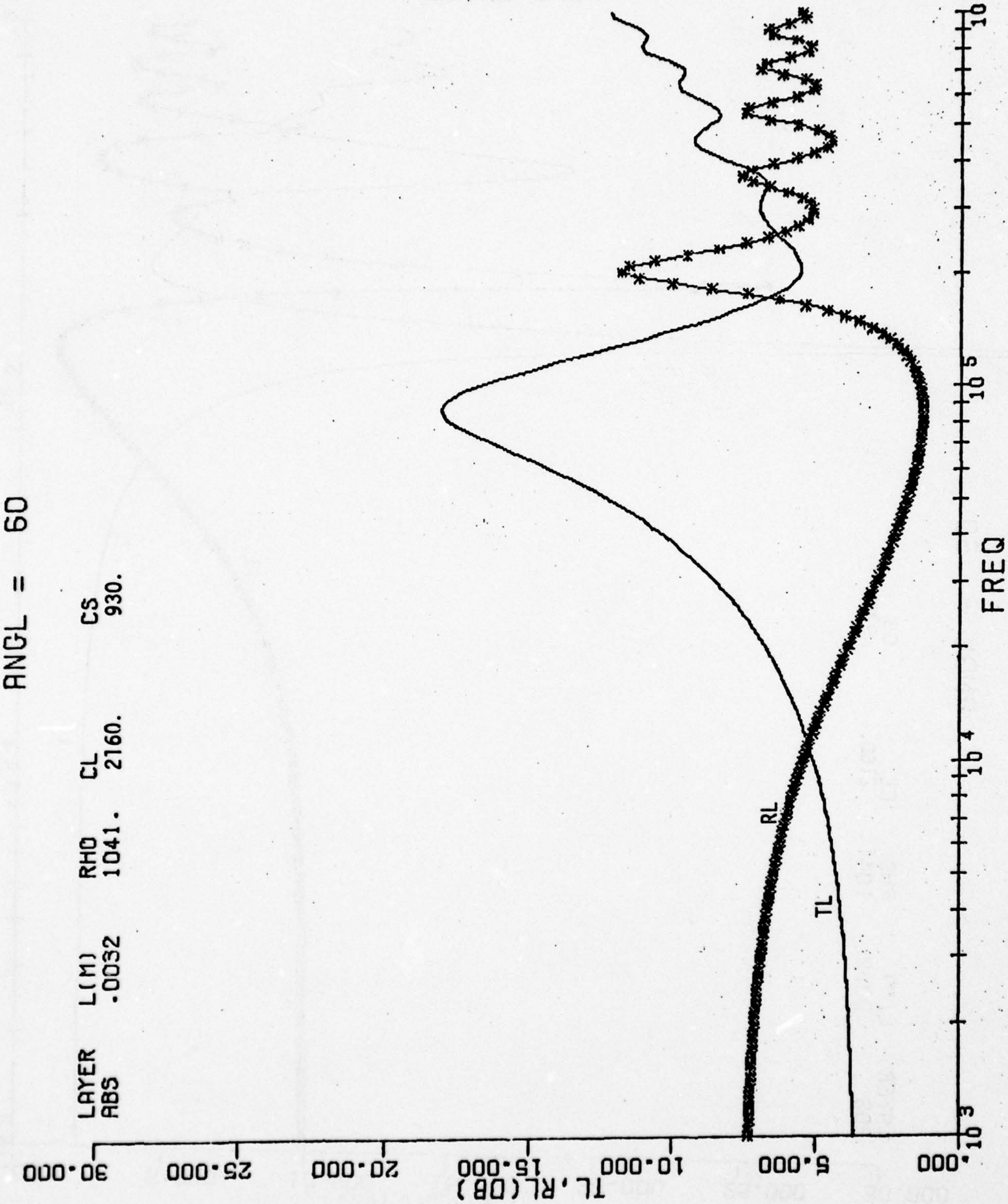


Figure 4(g). Transmission and Reflection Loss vs Frequency for Water/ABS/FC-75

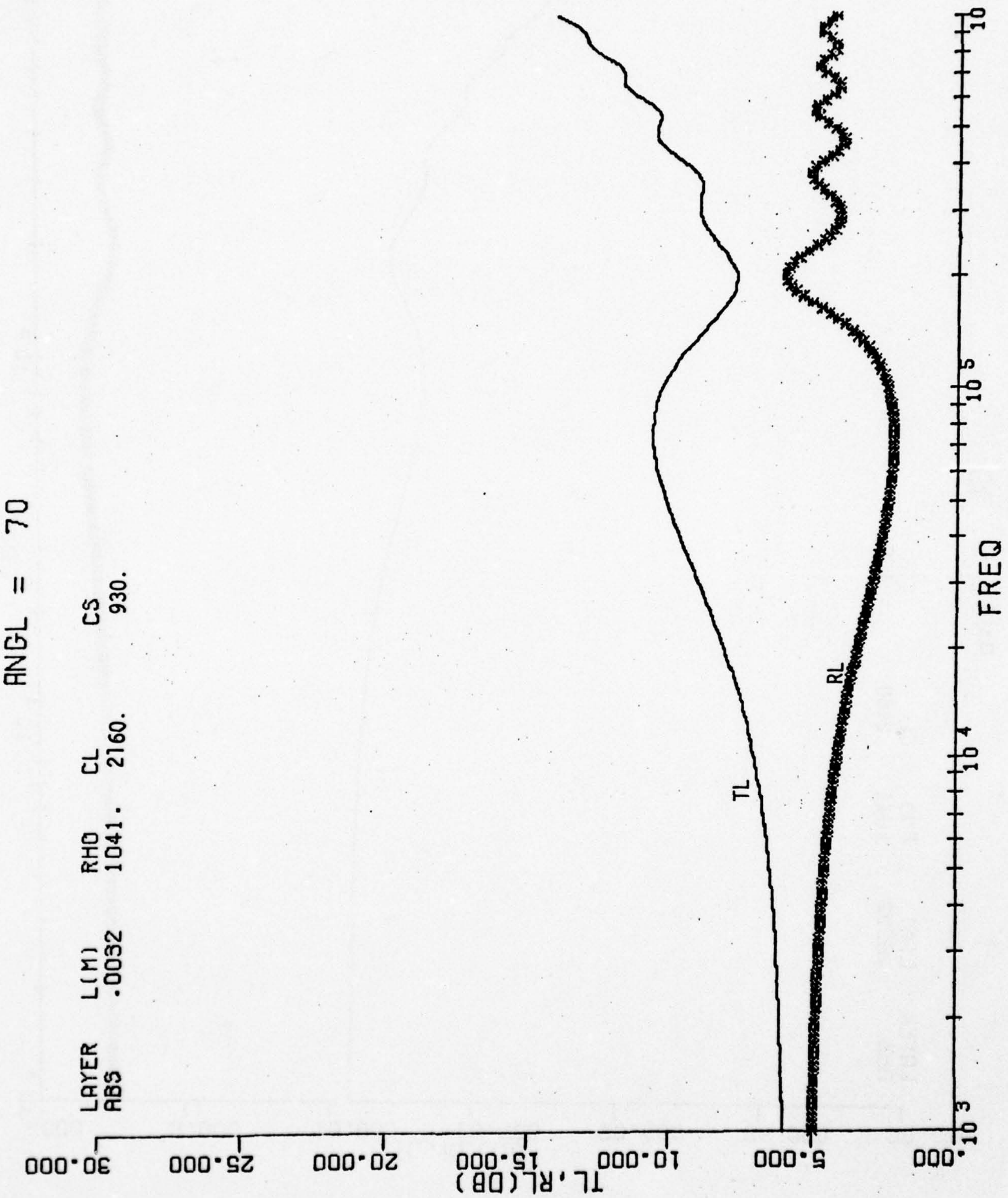


Figure 4(h). Transmission and Reflection Loss
vs Frequency for Water/ABS/FC-75

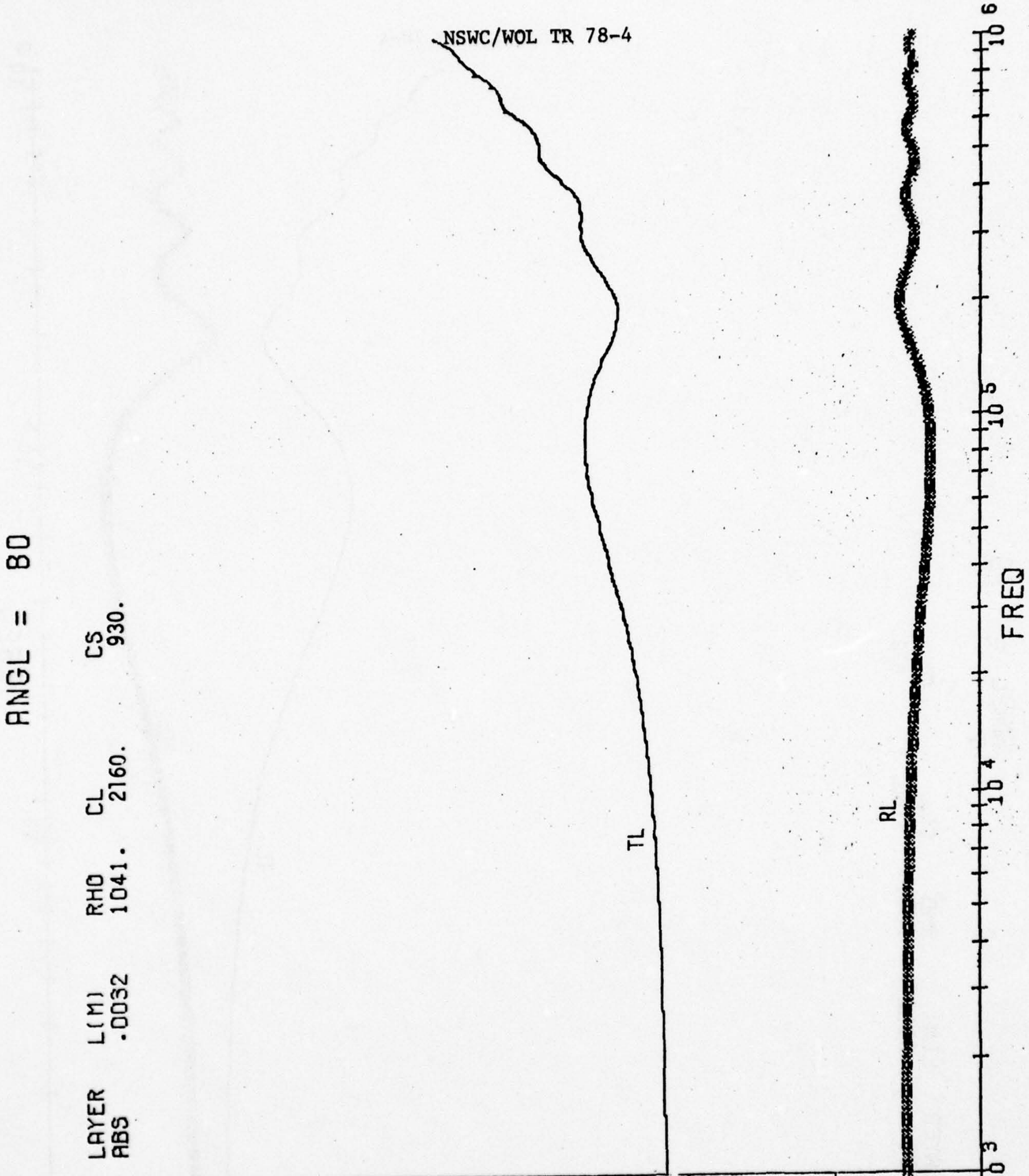


Figure 4(i). Transmission and Reflection Loss vs Frequency for Water/ABS/FC-75

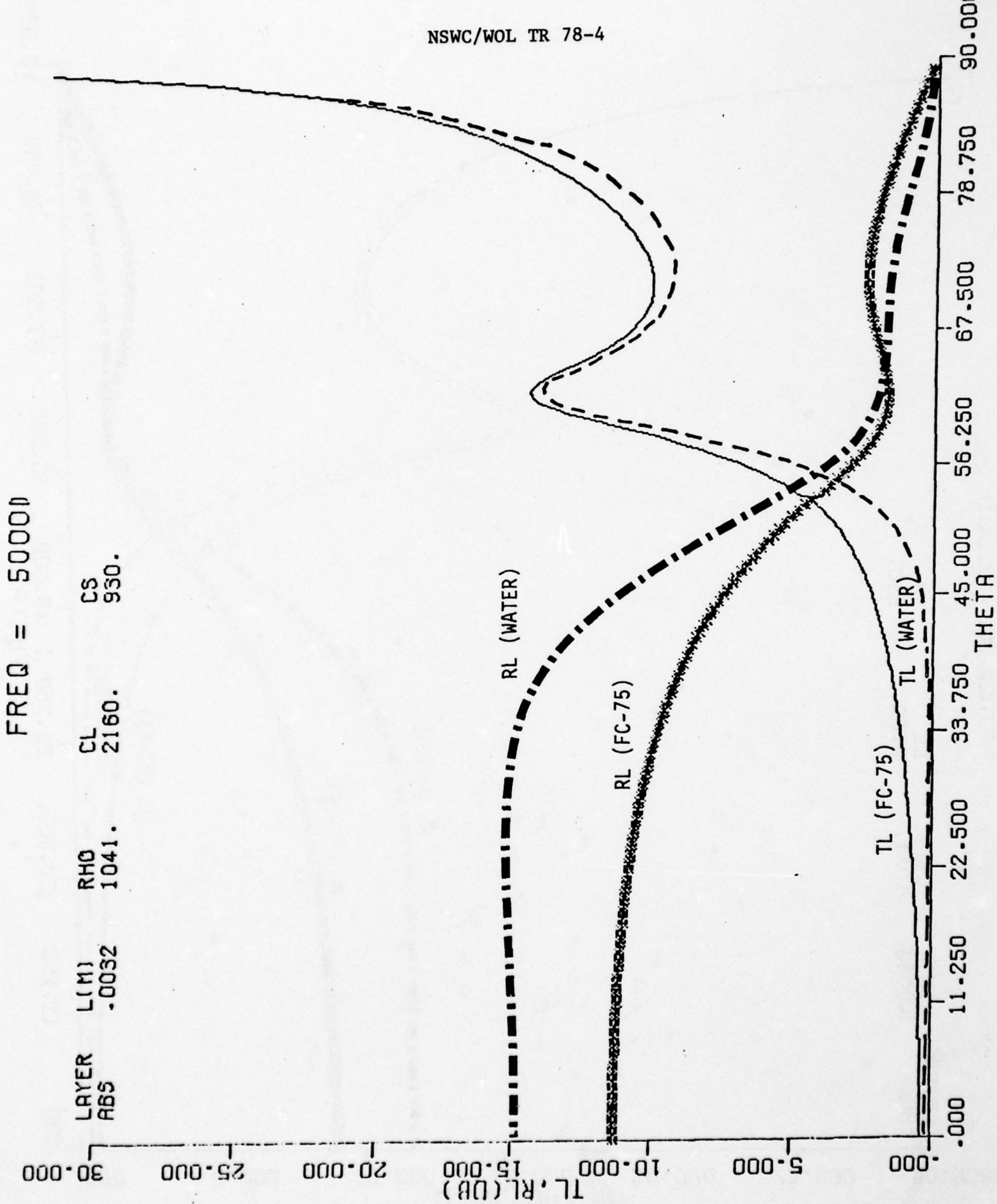


Figure 5(a). Transmission and Reflection Loss vs θ for Water/ABS/FC-75 and Water/ABS/Water

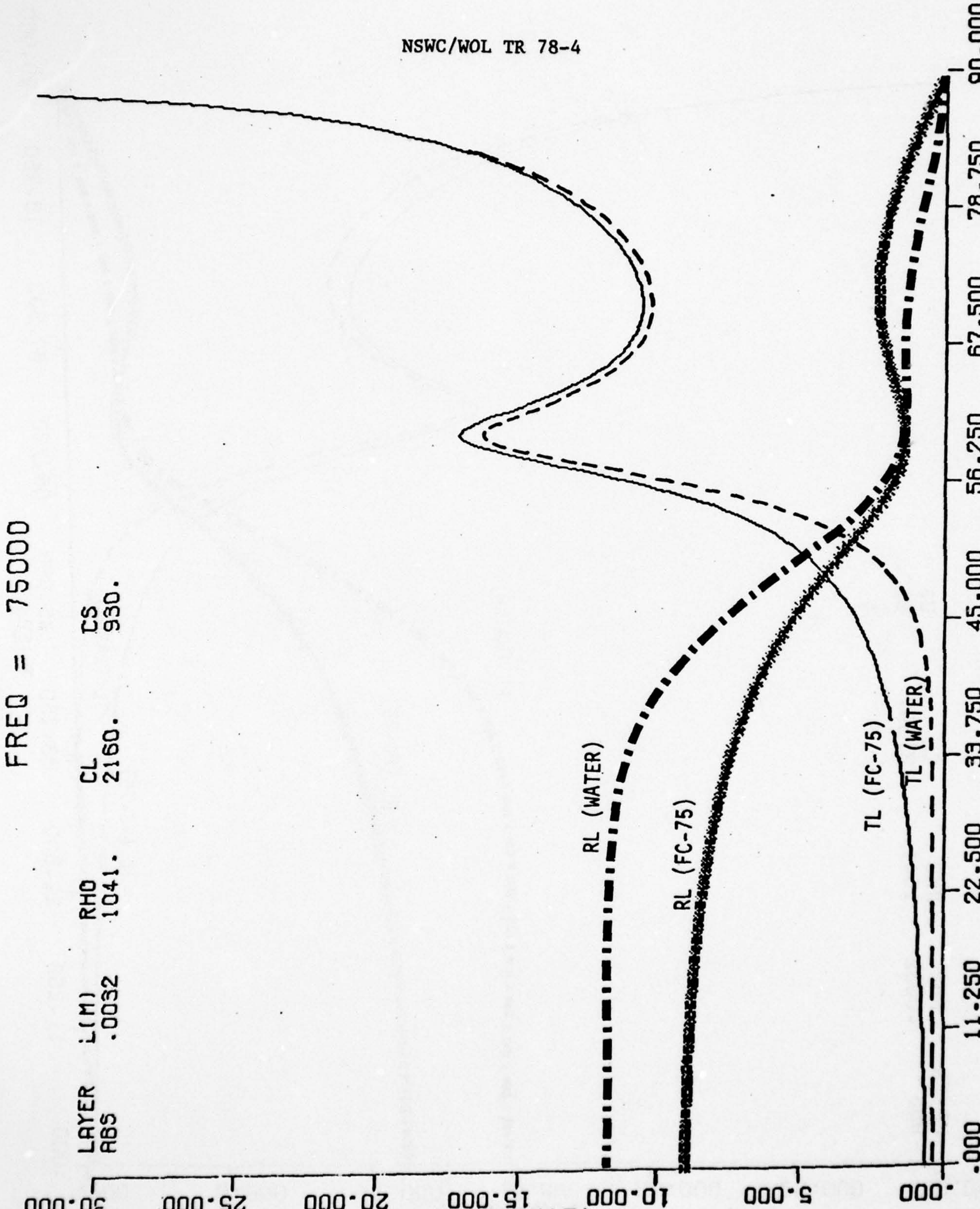


Figure 5(b). Transmission and Reflection Loss vs θ for Water/ABS/FC-75 and Water/ABS/Water

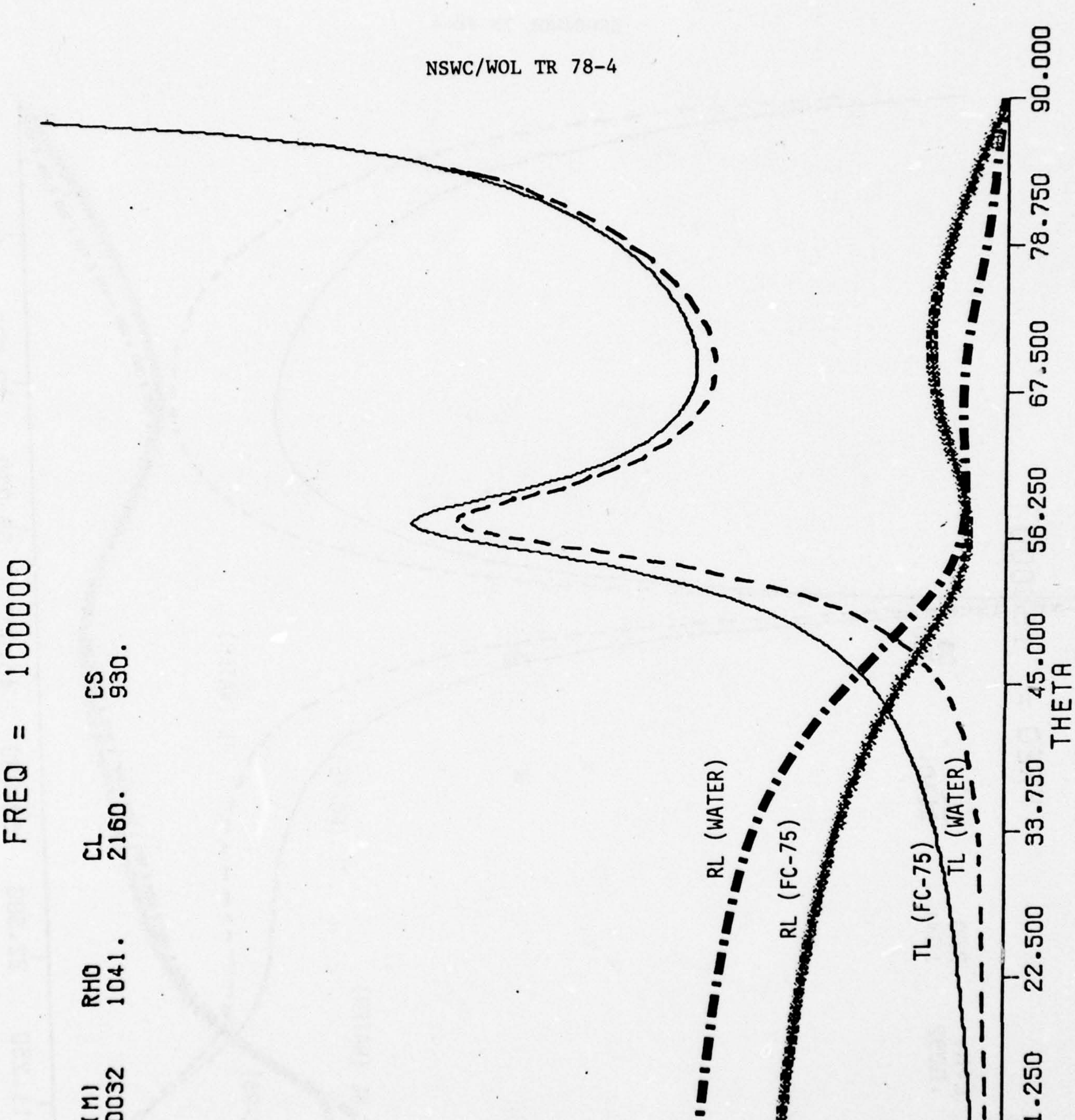


Figure 5(c). Transmission and Reflection Loss vs θ for Water/ABS/FC-75 and Water/ABS/Water

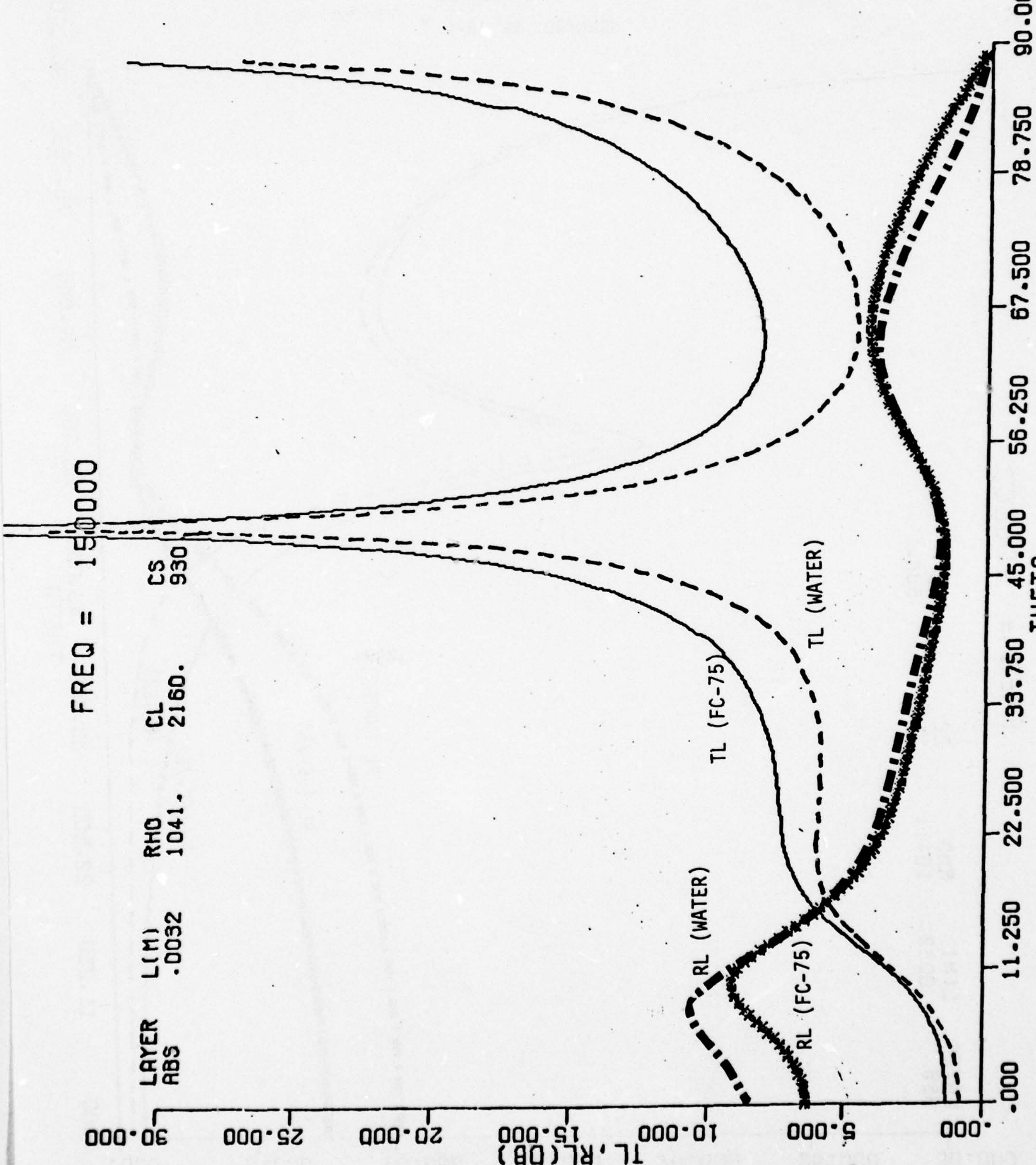


Figure 5(d). Transmission and Reflection Loss vs θ for Water/ABS/FC-75 and Water/ABS/Water

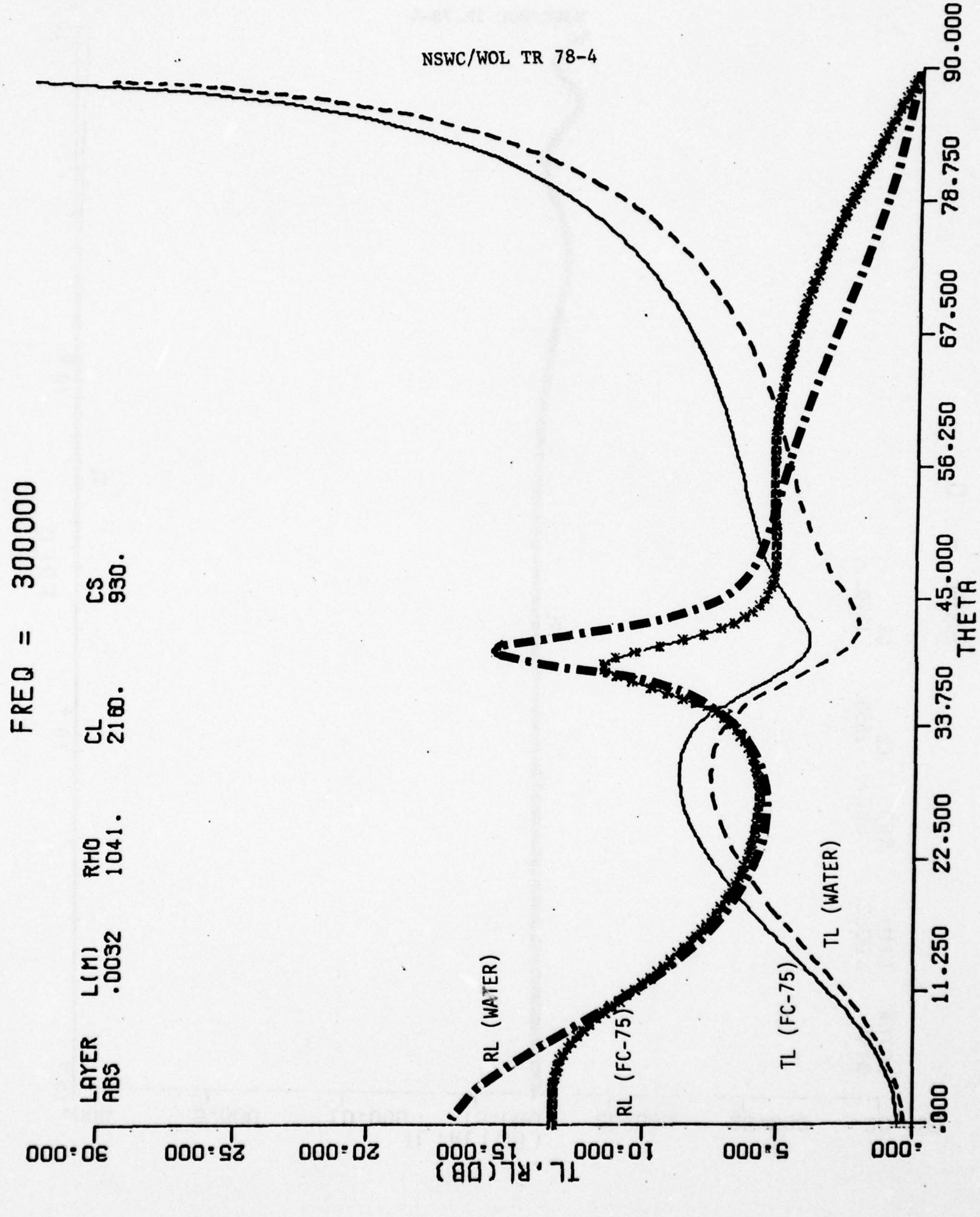


Figure 5(e). Transmission and Reflection Loss vs θ for Water/ABS/FC-75 and Water/ABS/Water

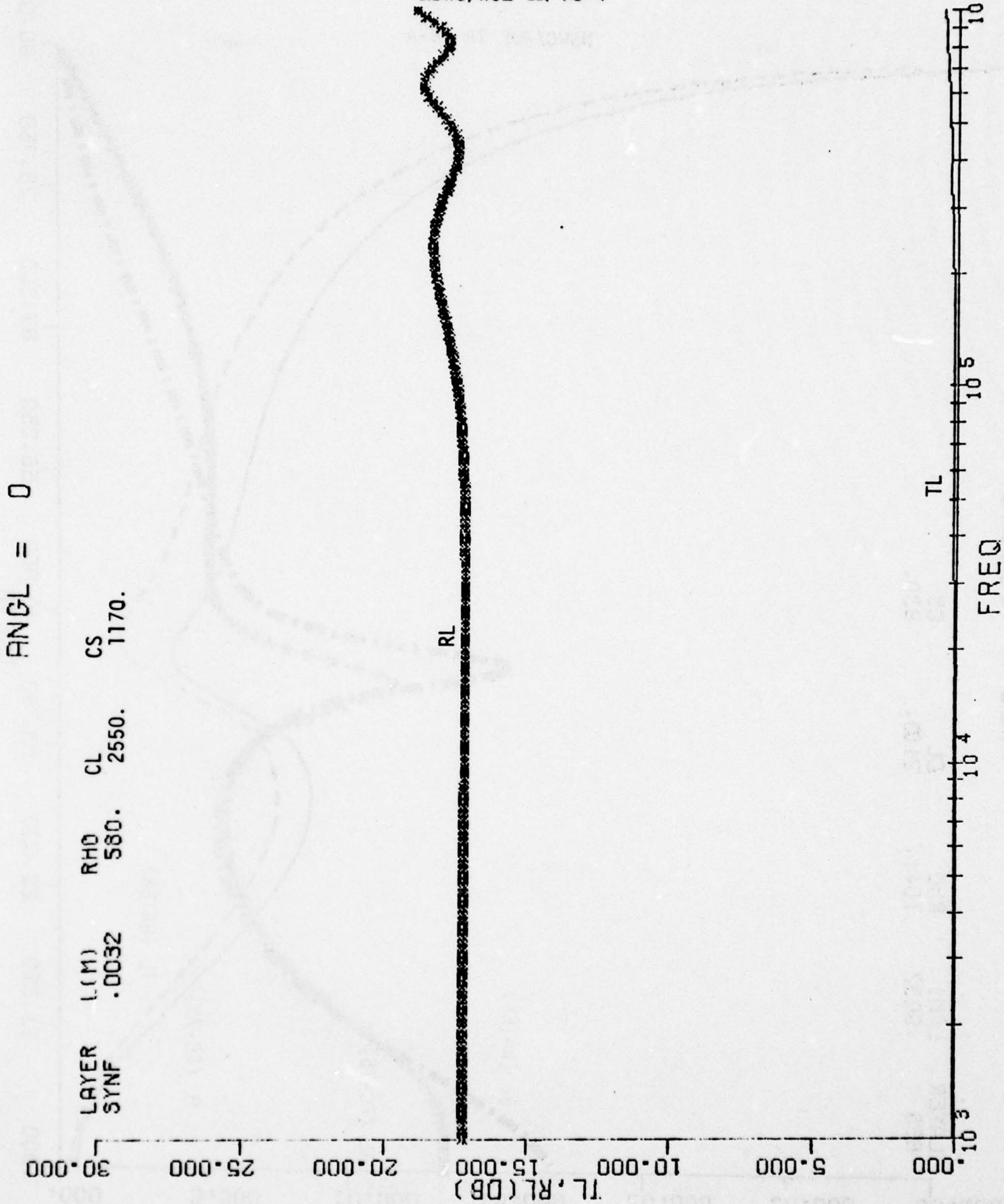


Figure 6(a). Transmission and Reflection Loss vs Frequency for Water/Syntactic Foam/FC-75

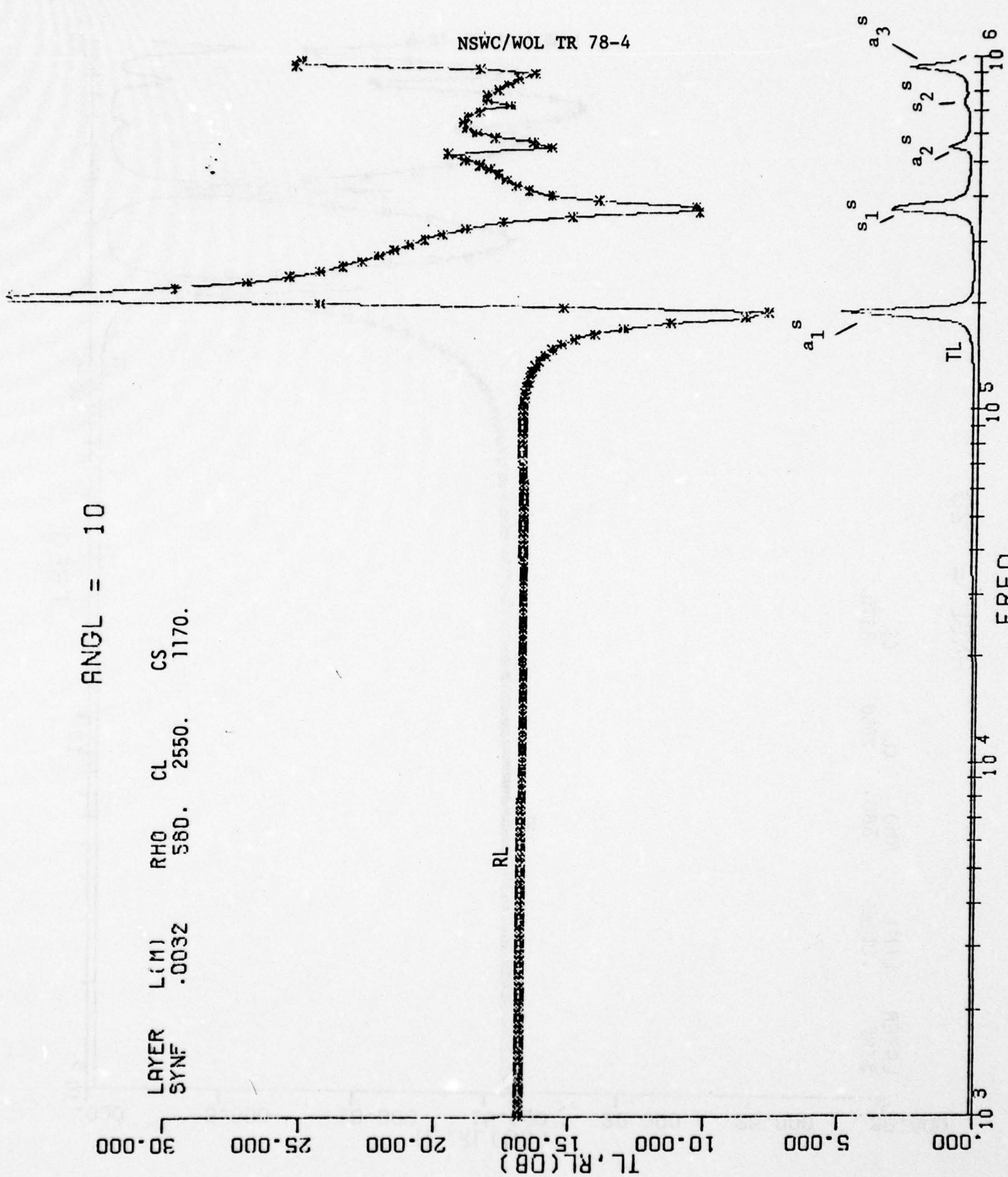


Figure 6(b). Transmission and Reflection Loss vs Frequency for Water/Syntactic Foam/FC-75

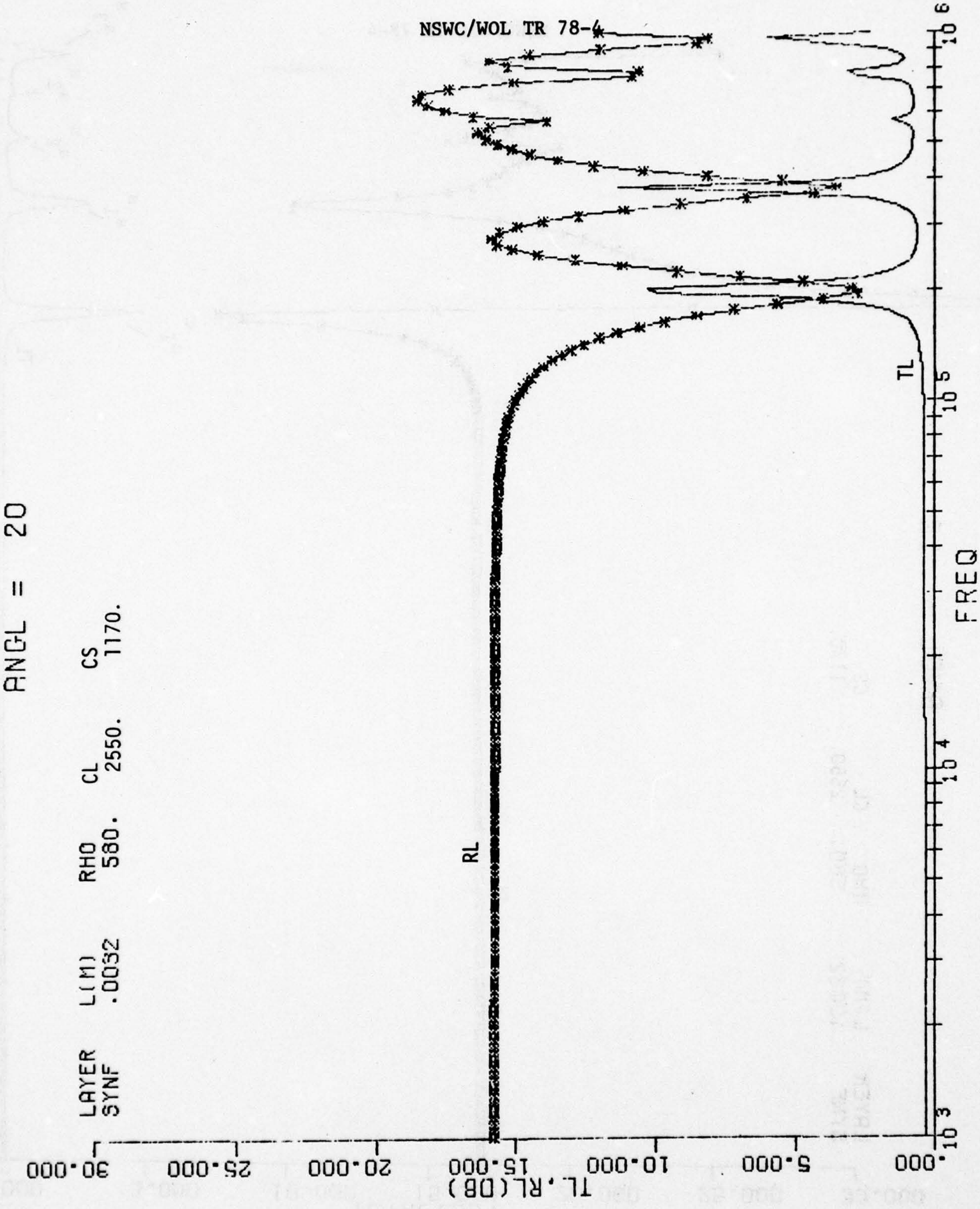


Figure 6(c). Transmission and Reflection Loss vs Frequency for Water/Syntactic Foam/FC-75

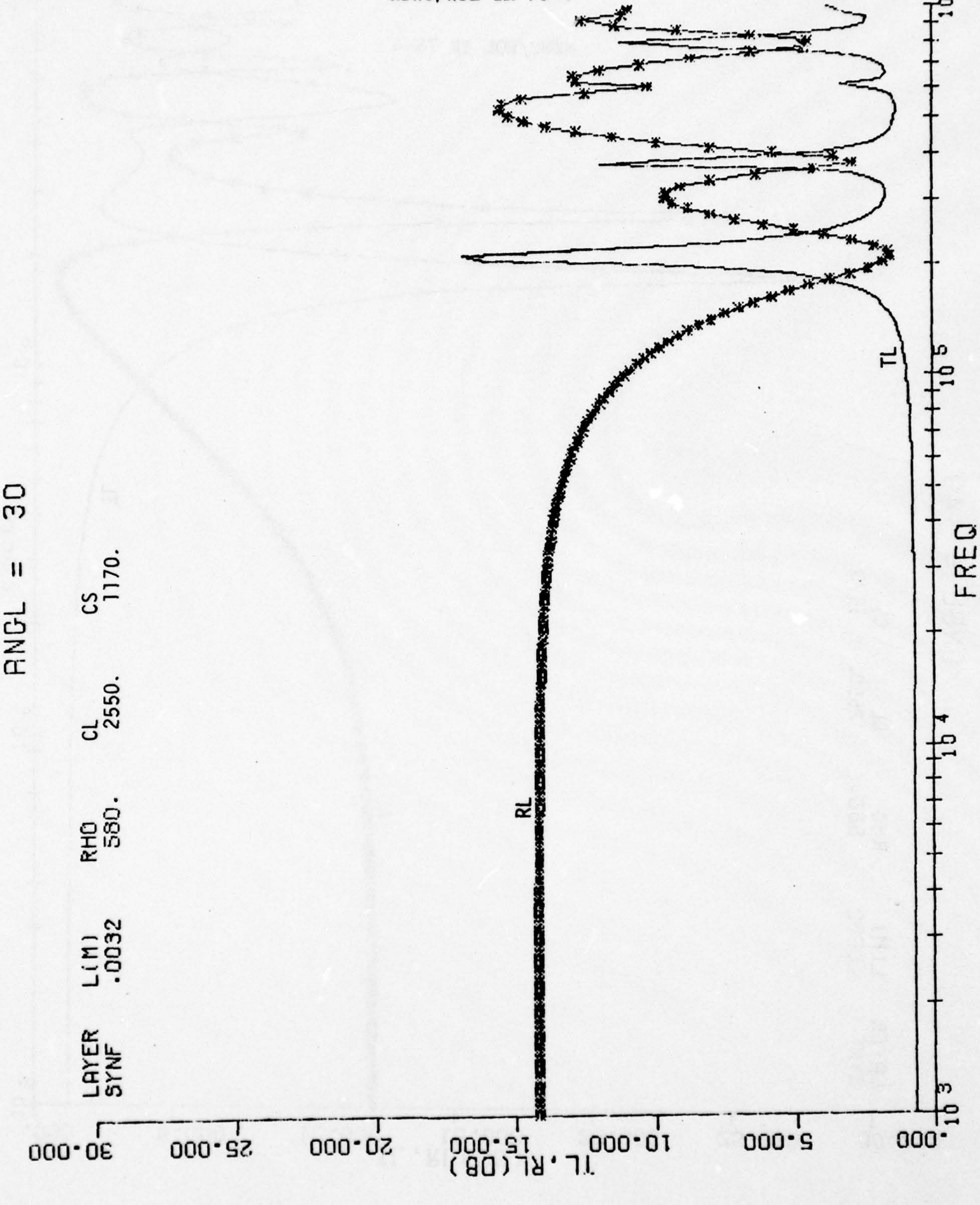


Figure 6(d). Transmission and Reflection Loss vs Frequency for Water/Syntactic Foam/FC-75

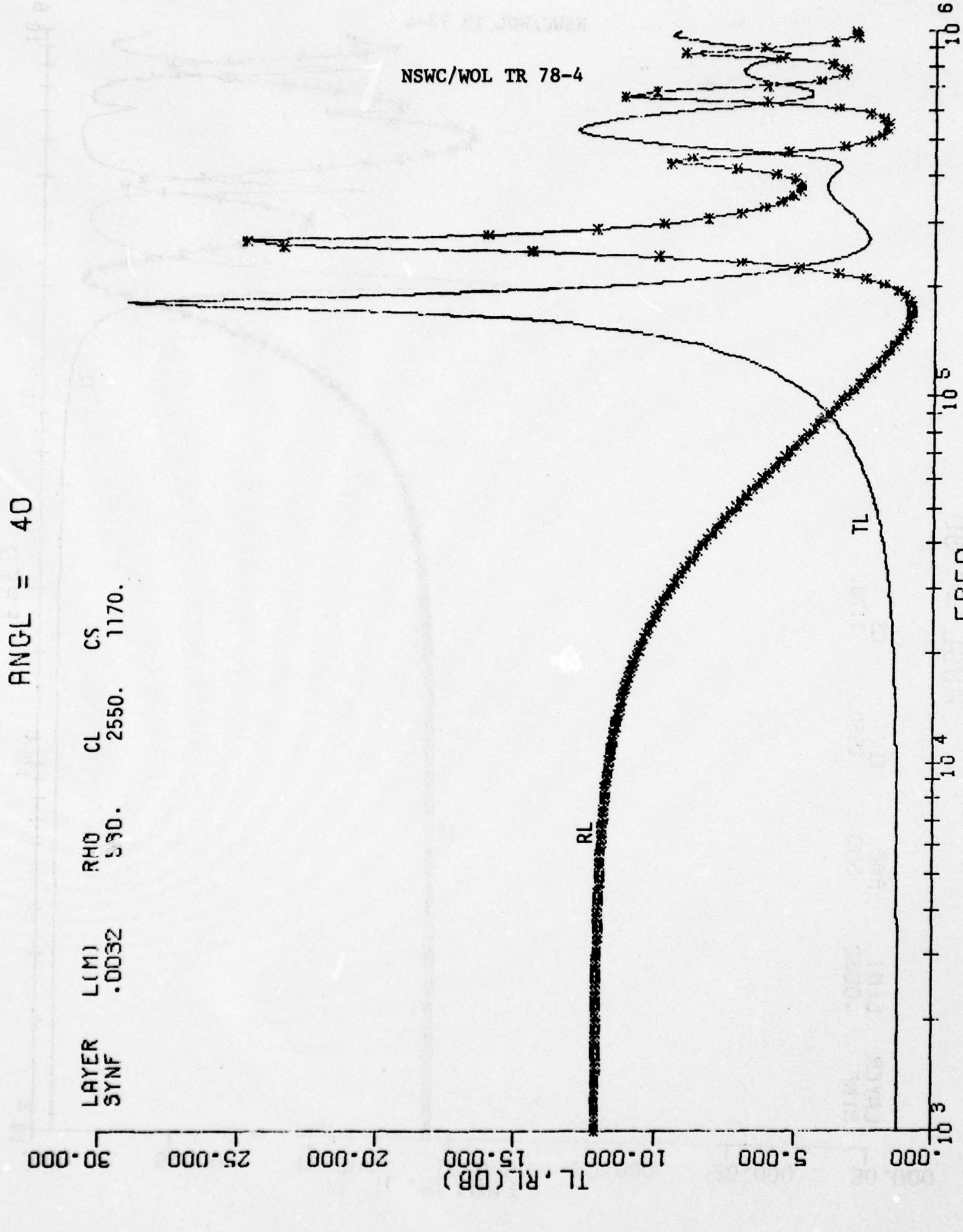


Figure 6(e). Transmission and Reflection Loss
vs Frequency for Water/Syntactic Foam/FC-75

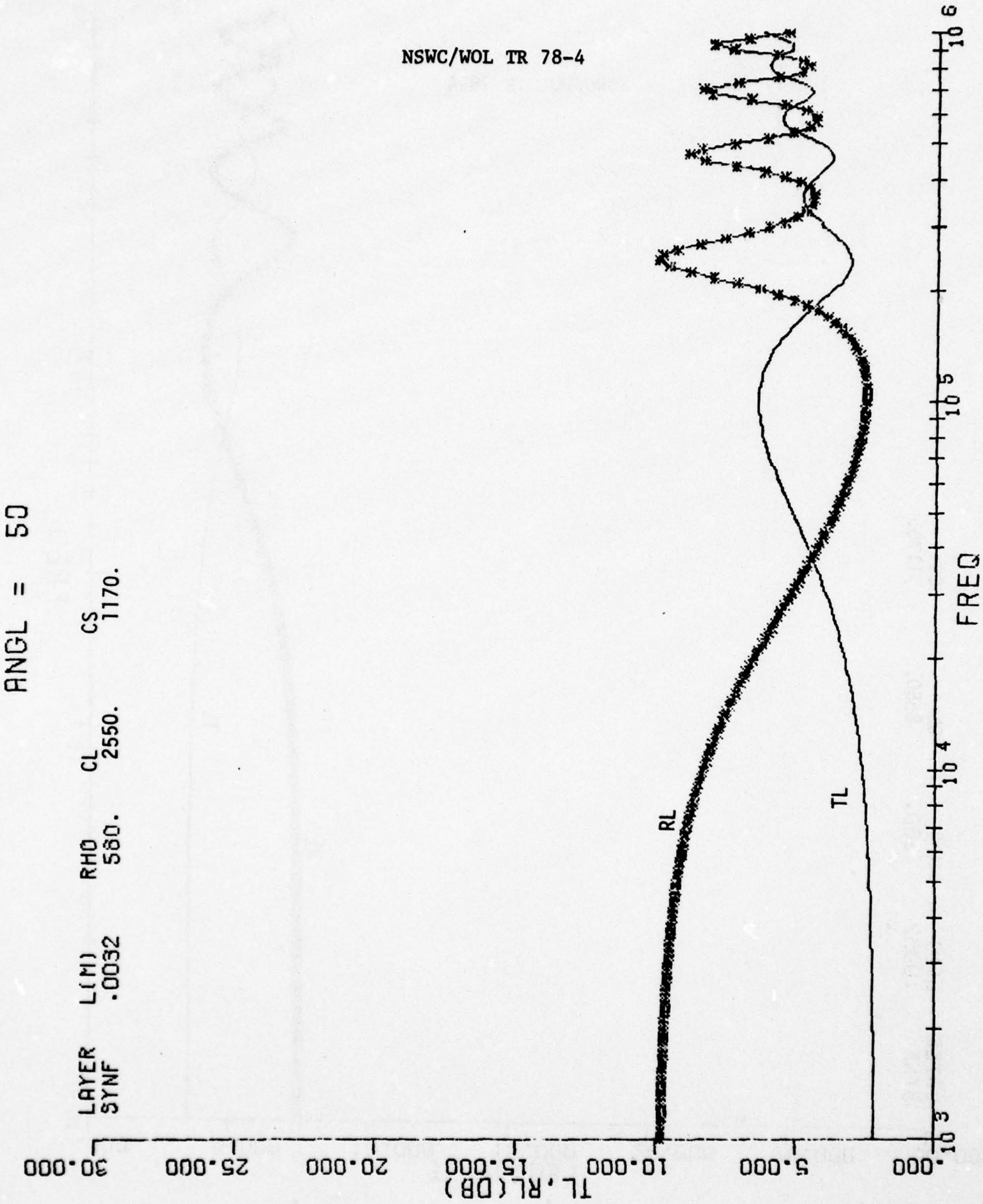


Figure 6(f). Transmission and Reflection Loss
vs Frequency for Water/Syntactic Foam/FC-75

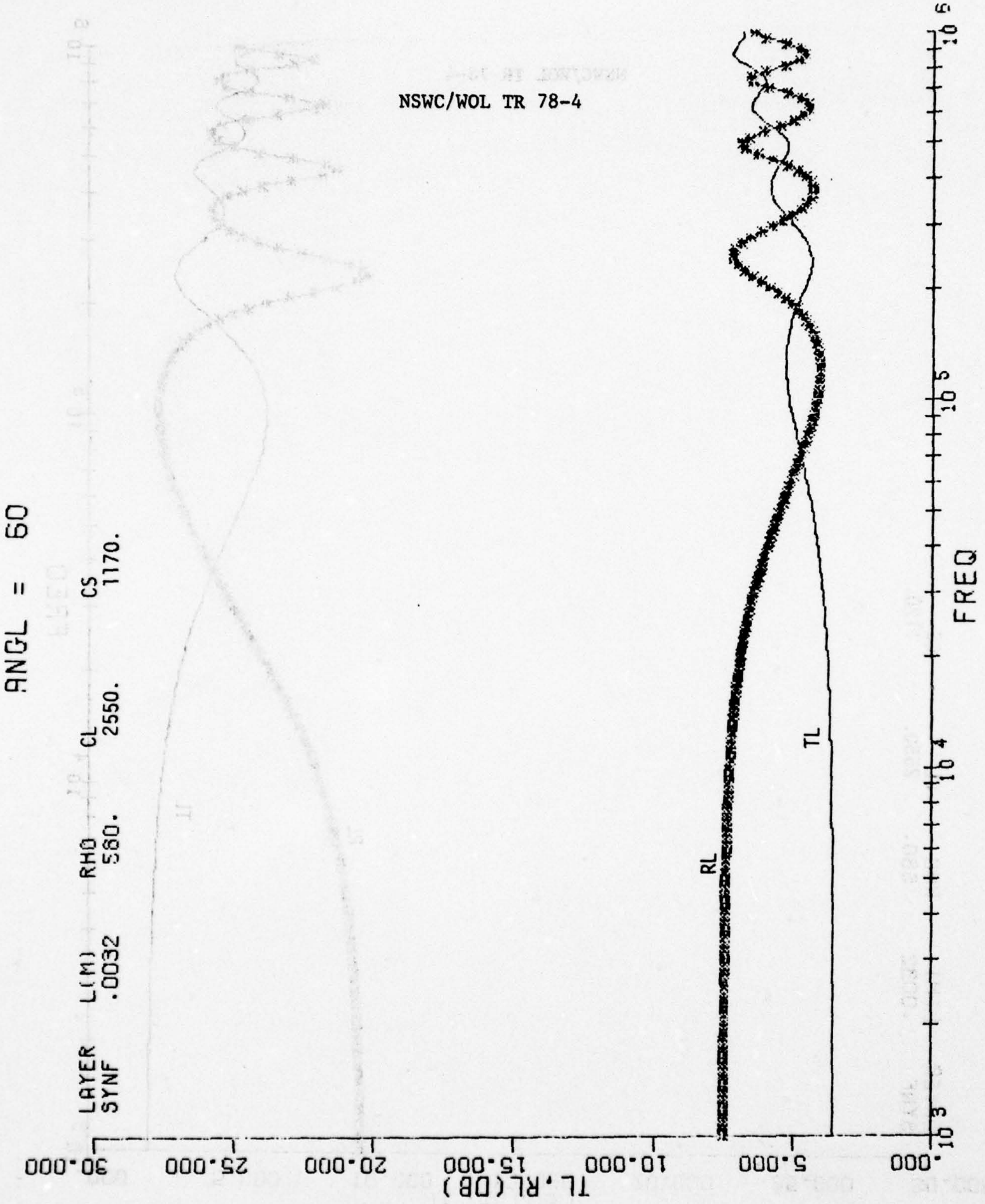


Figure 6(g). Transmission and Reflection Loss
vs Frequency for Water/Syntactic Foam/FC-75

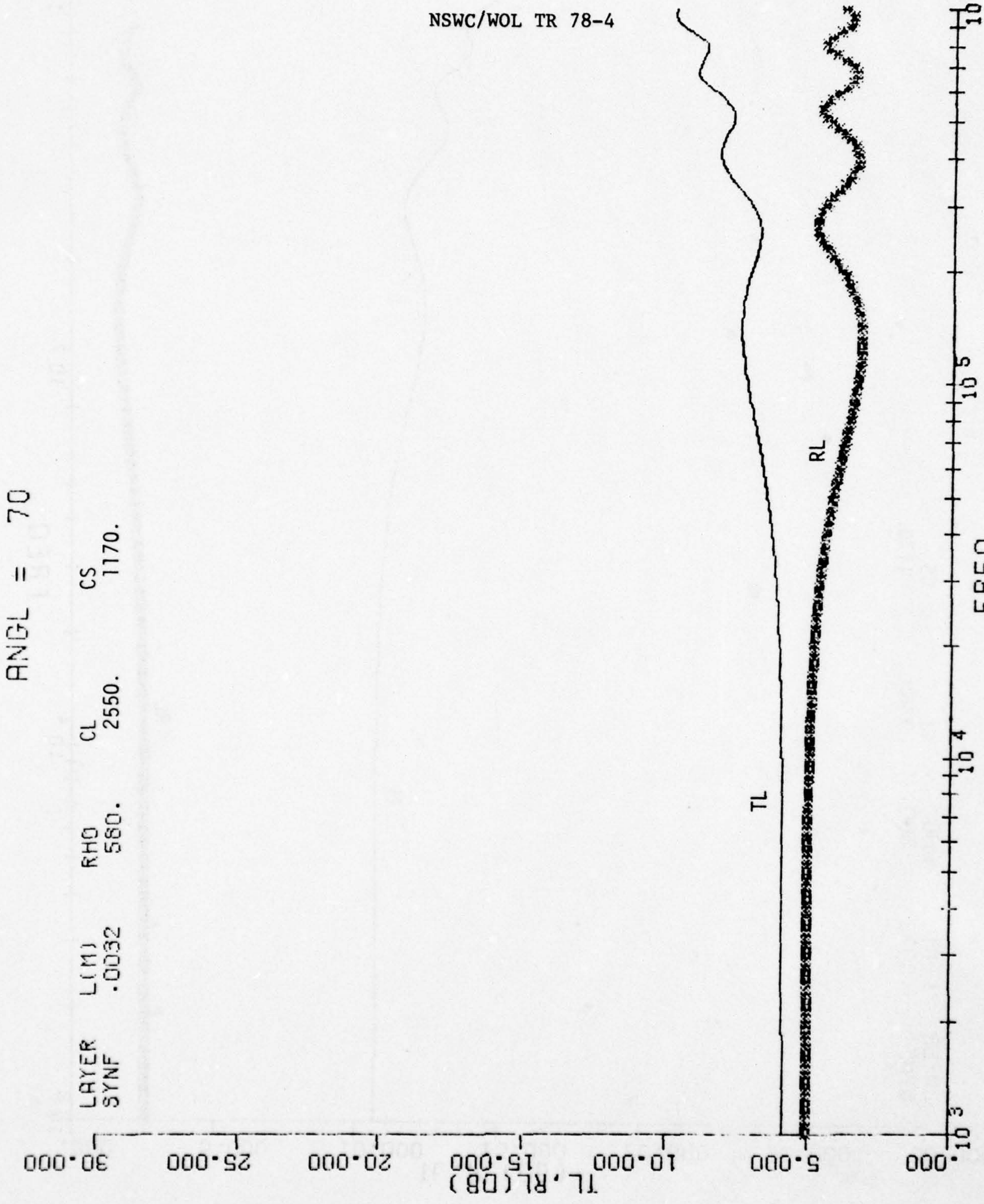


Figure 6(h). Transmission and Reflection Loss vs Frequency for Water/Syntactic Foam/FC-75

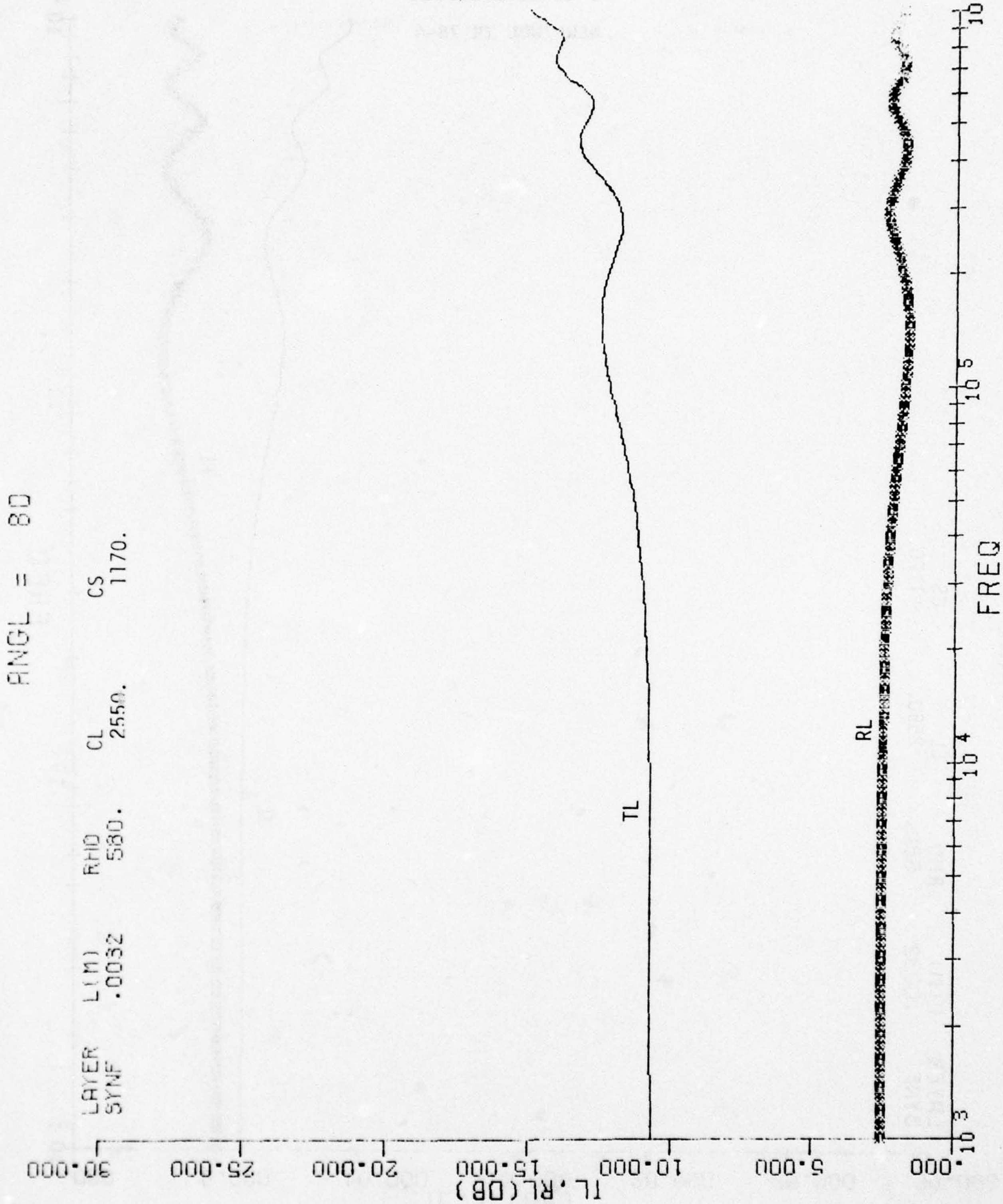


Figure 6(i). Transmission and Reflection Loss vs Frequency for Water/Syntactic Foam/FC-75

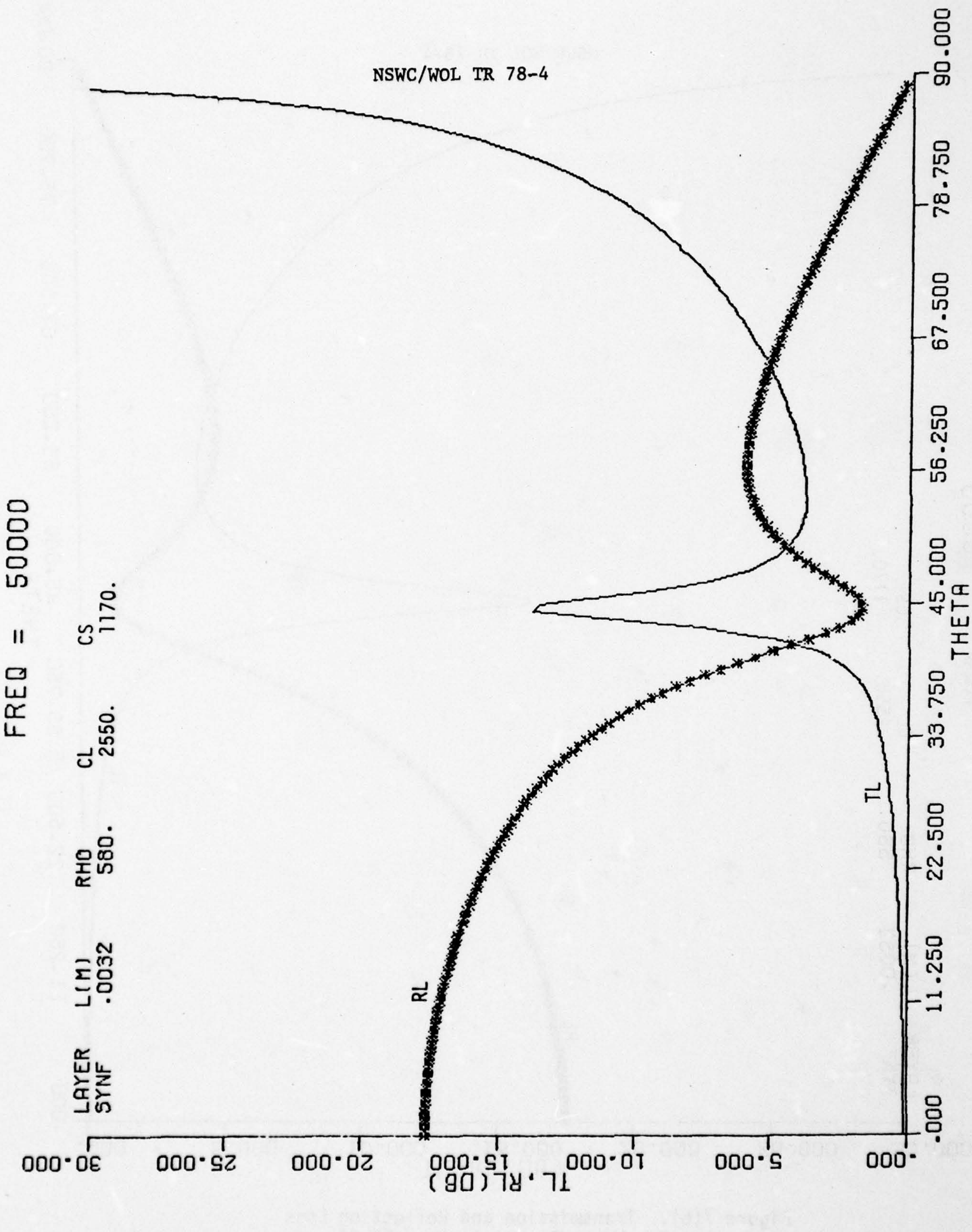


Figure 7(a). Transmission and Reflection Loss vs θ for Water/Syntactic Foam/FC-75

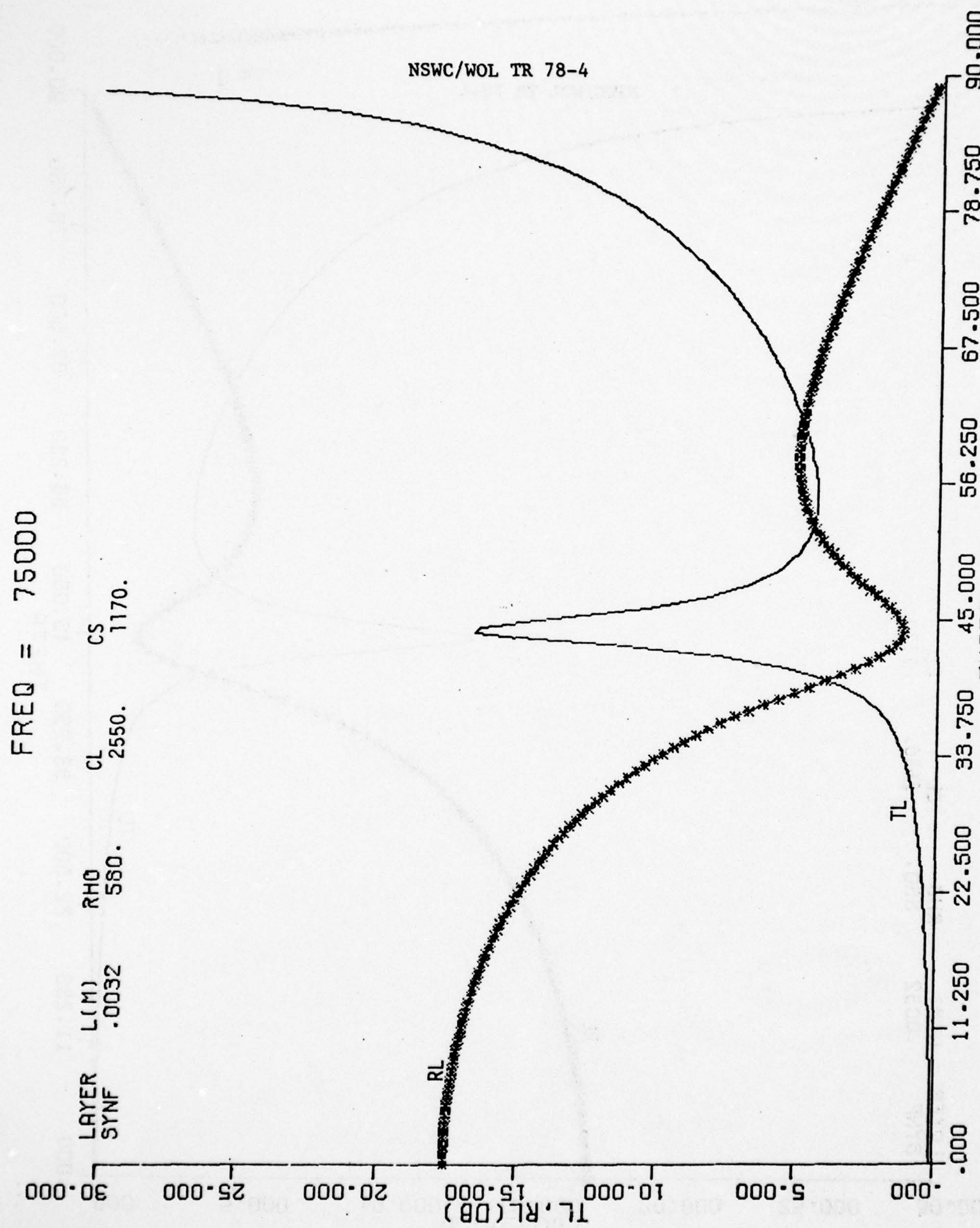


Figure 7(b). Transmission and Reflection Loss
vs θ for Water/Syntactic Foam/FC-75

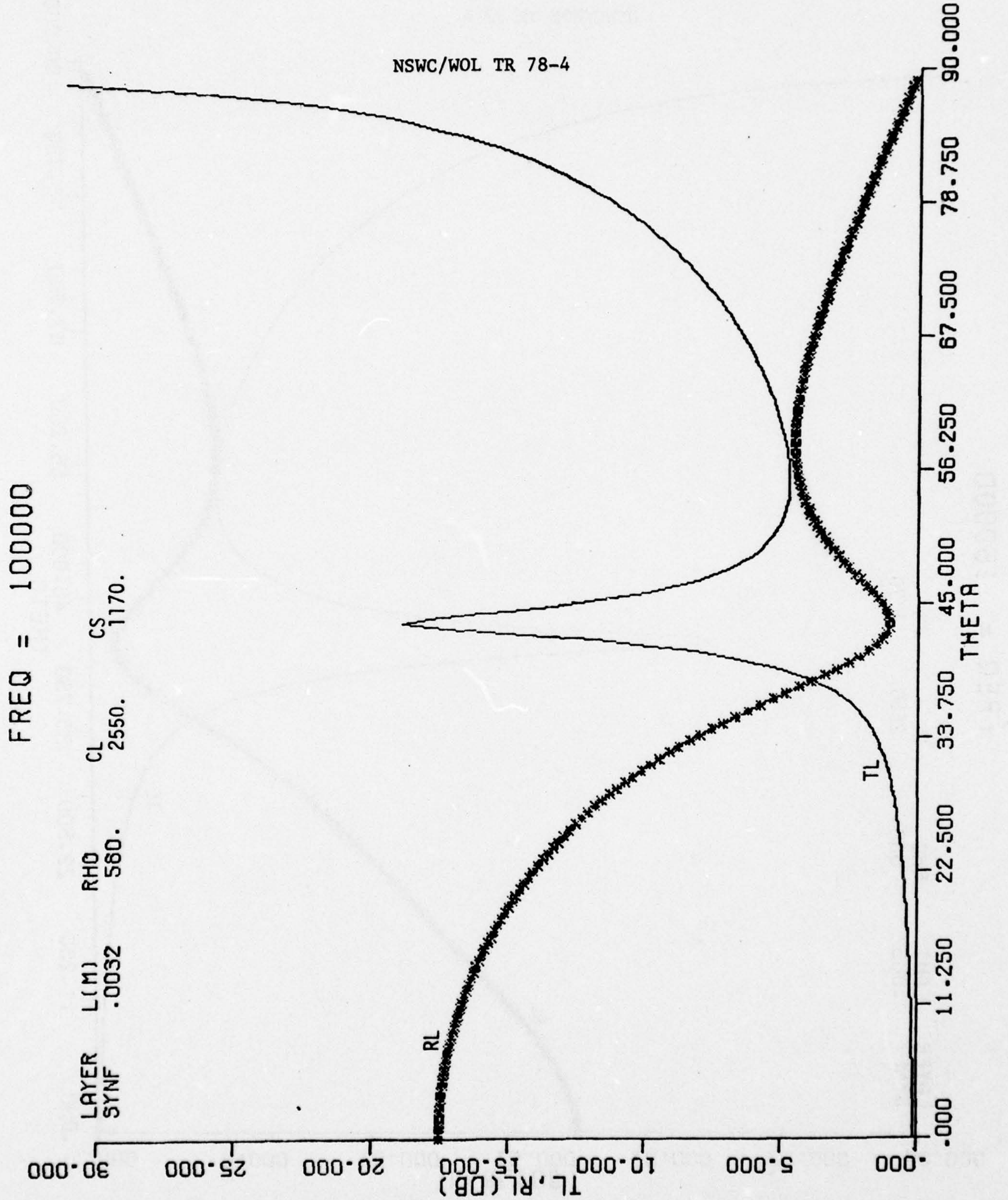


Figure 7(c). Transmission and Reflection Loss vs θ for Water/Syntactic Foam/FC-75

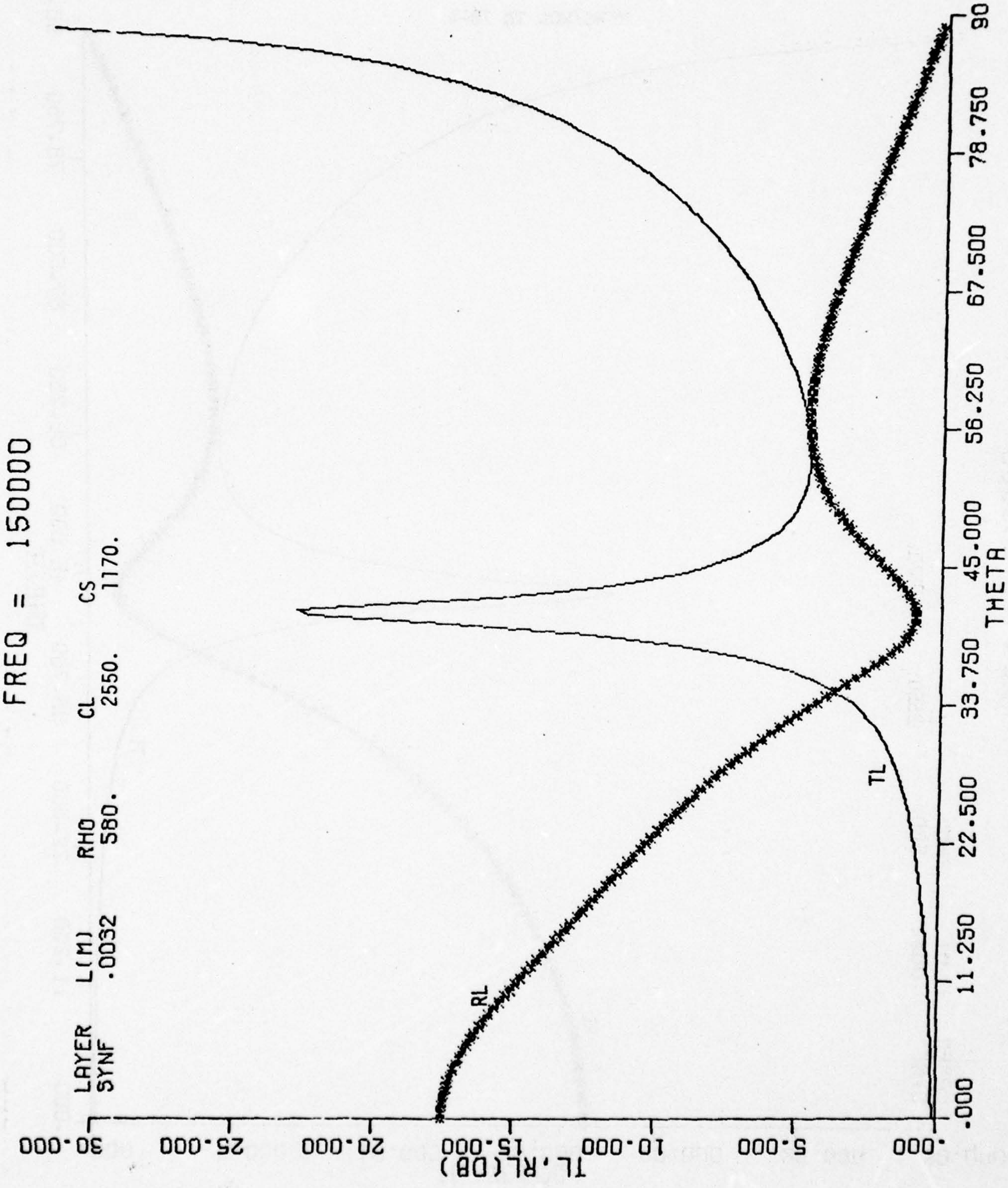


Figure 7(d). Transmission and Reflection Loss vs θ for Water/Syntactic Foam/FC-75

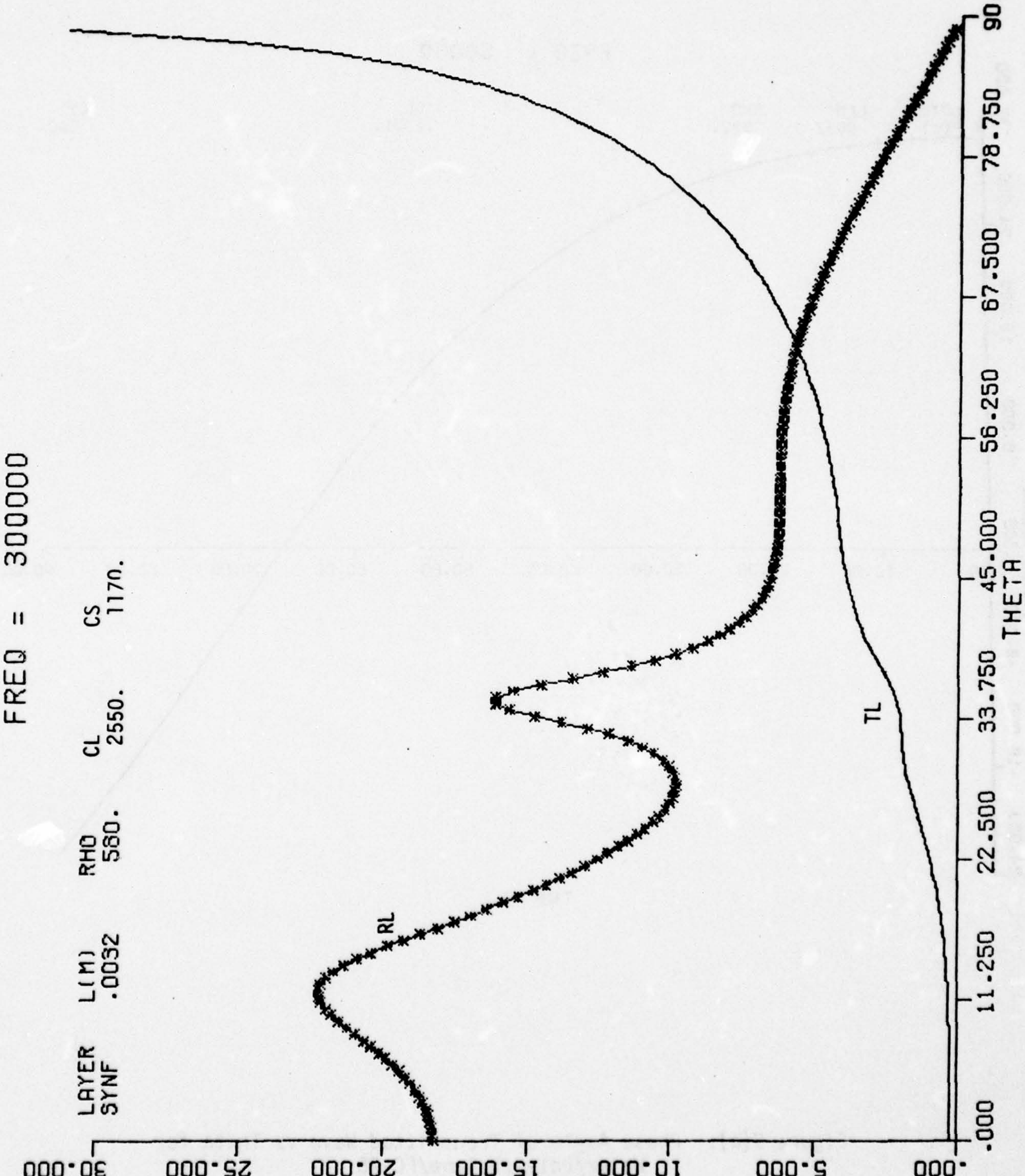


Figure 7(e). Transmission and Reflection Loss vs θ for Water/Syntactic Foam/FC-75

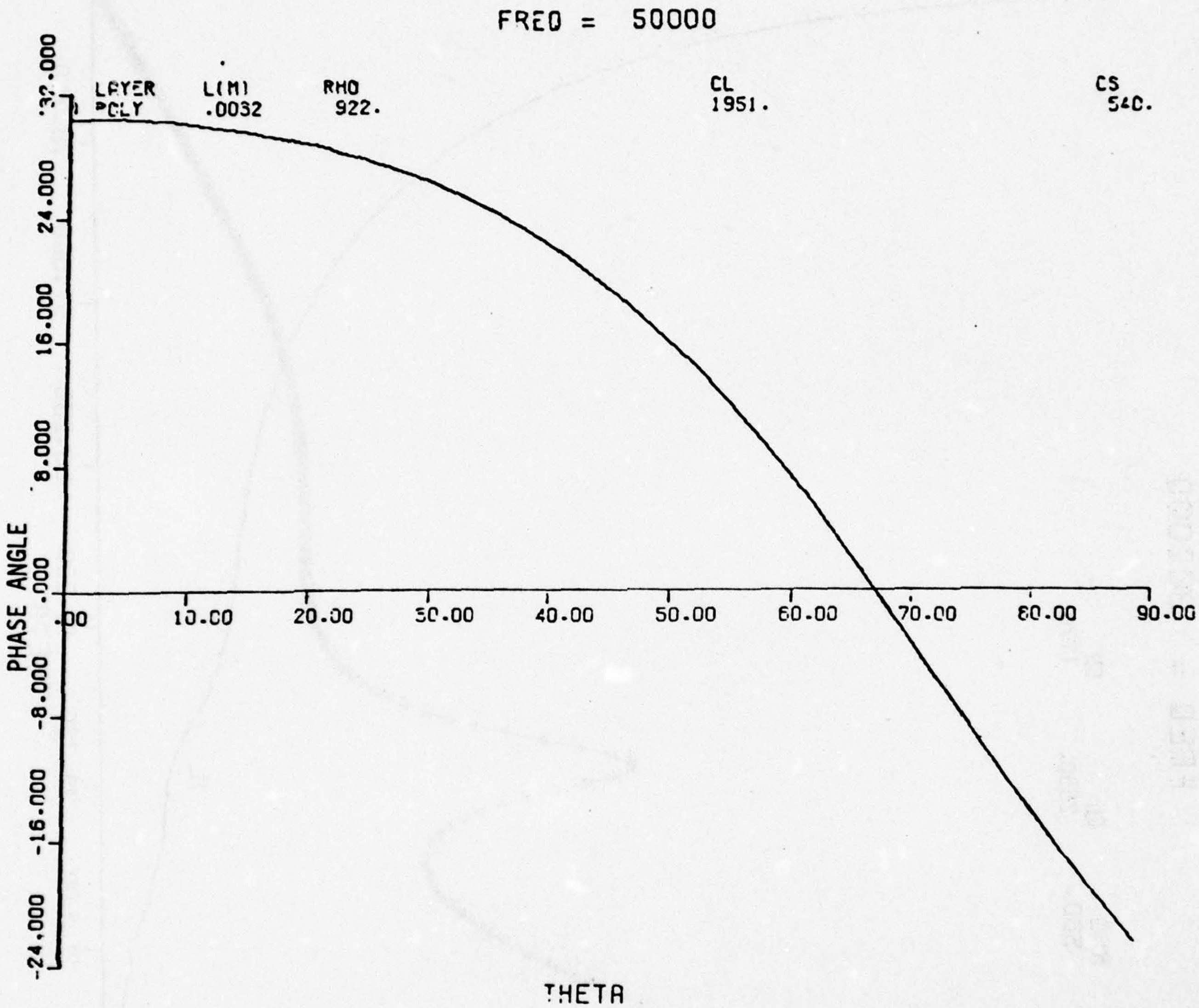


Figure 8(a). Phase Angle of Transmitted Wave vs Theta for Water/Polyethylene/FC-75

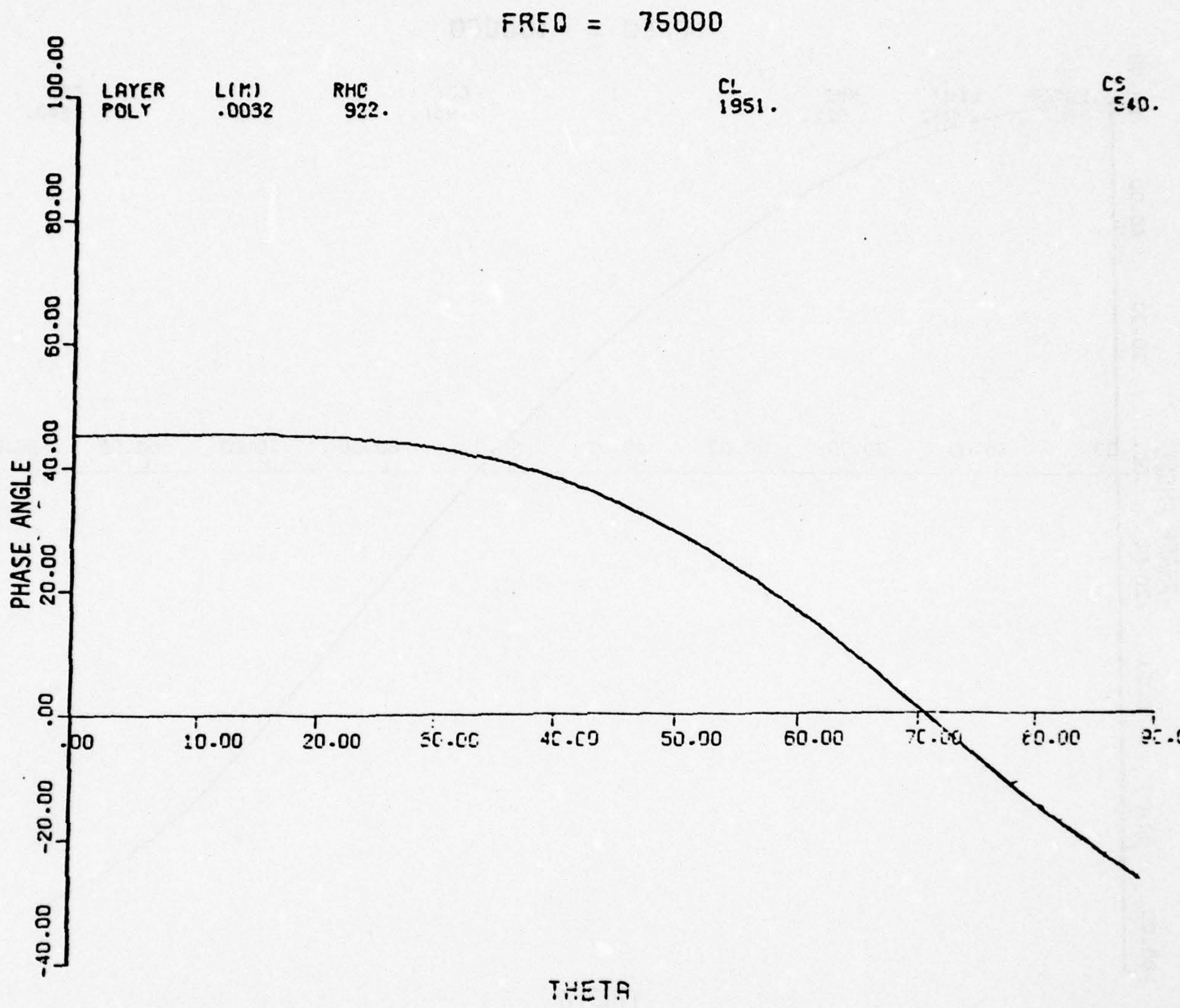


Figure 8(b). Phase Angle of Transmitted Wave vs Theta for Water/Polyethylene/FC-75

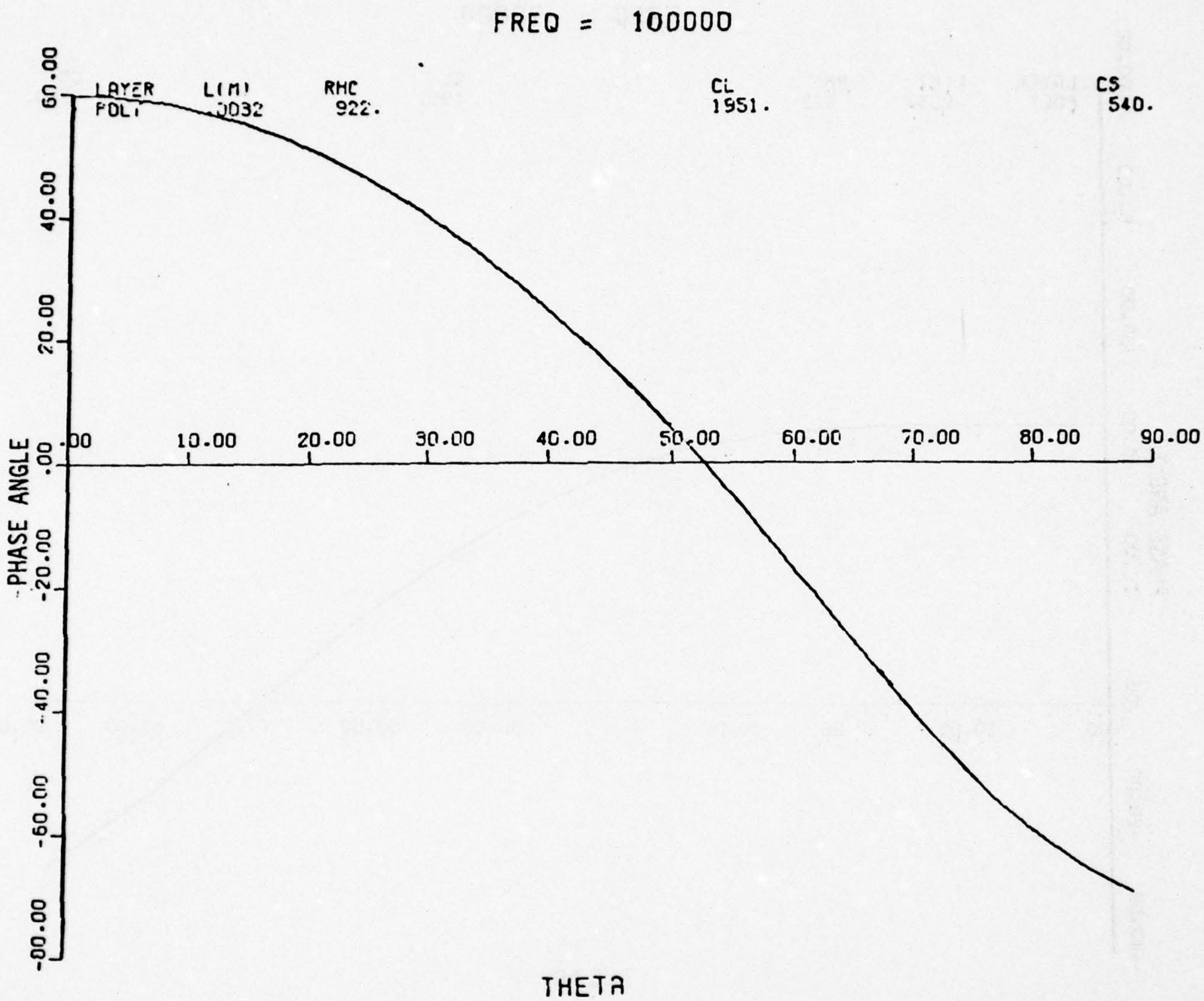


Figure 8(c). Phase Angle of Transmitted Wave vs Theta for Water/Polyethylene/FC-75

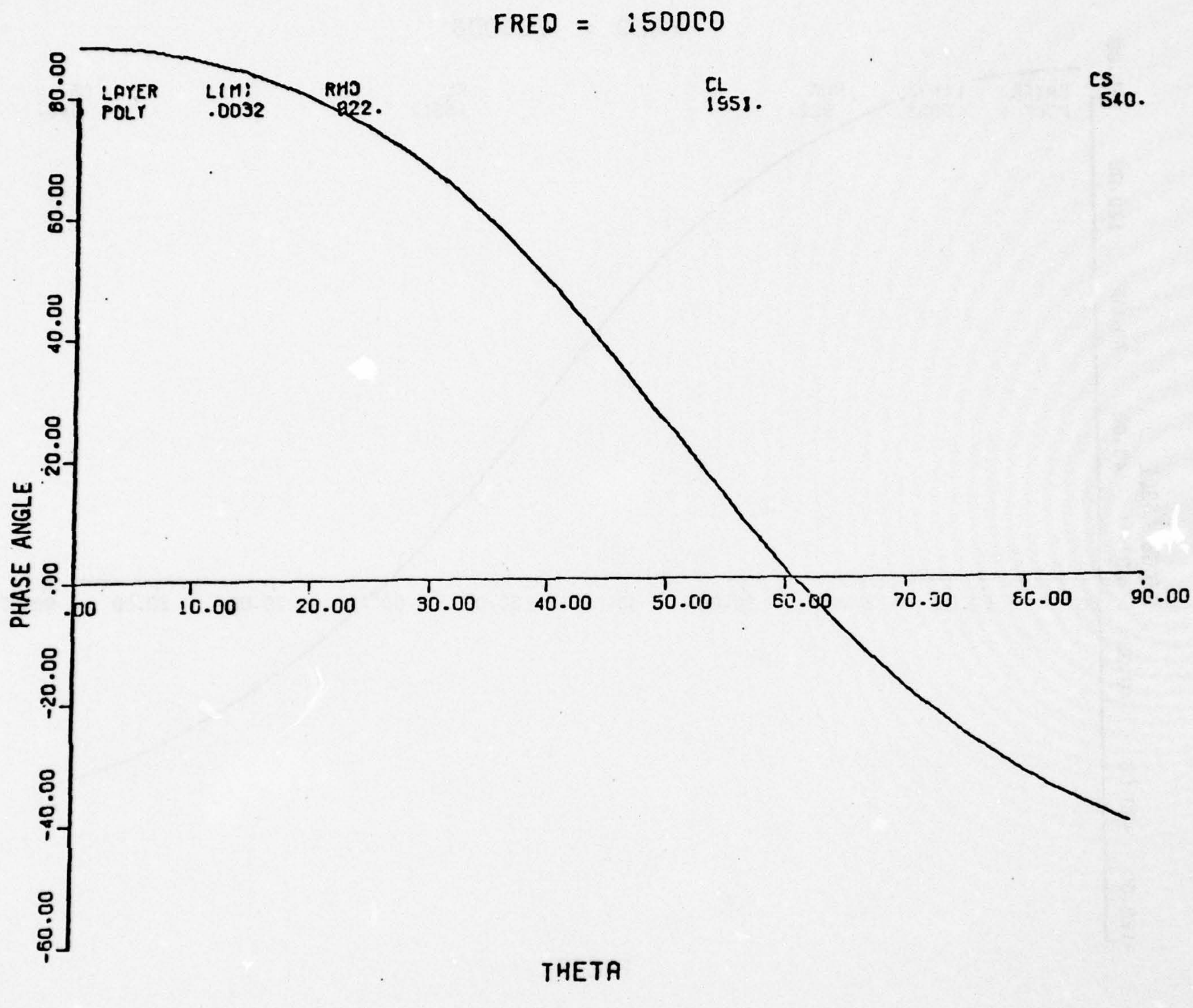


Figure 8(d). Phase Angle of Transmitted Wave vs Theta for Water/Polyethylene/FC-75

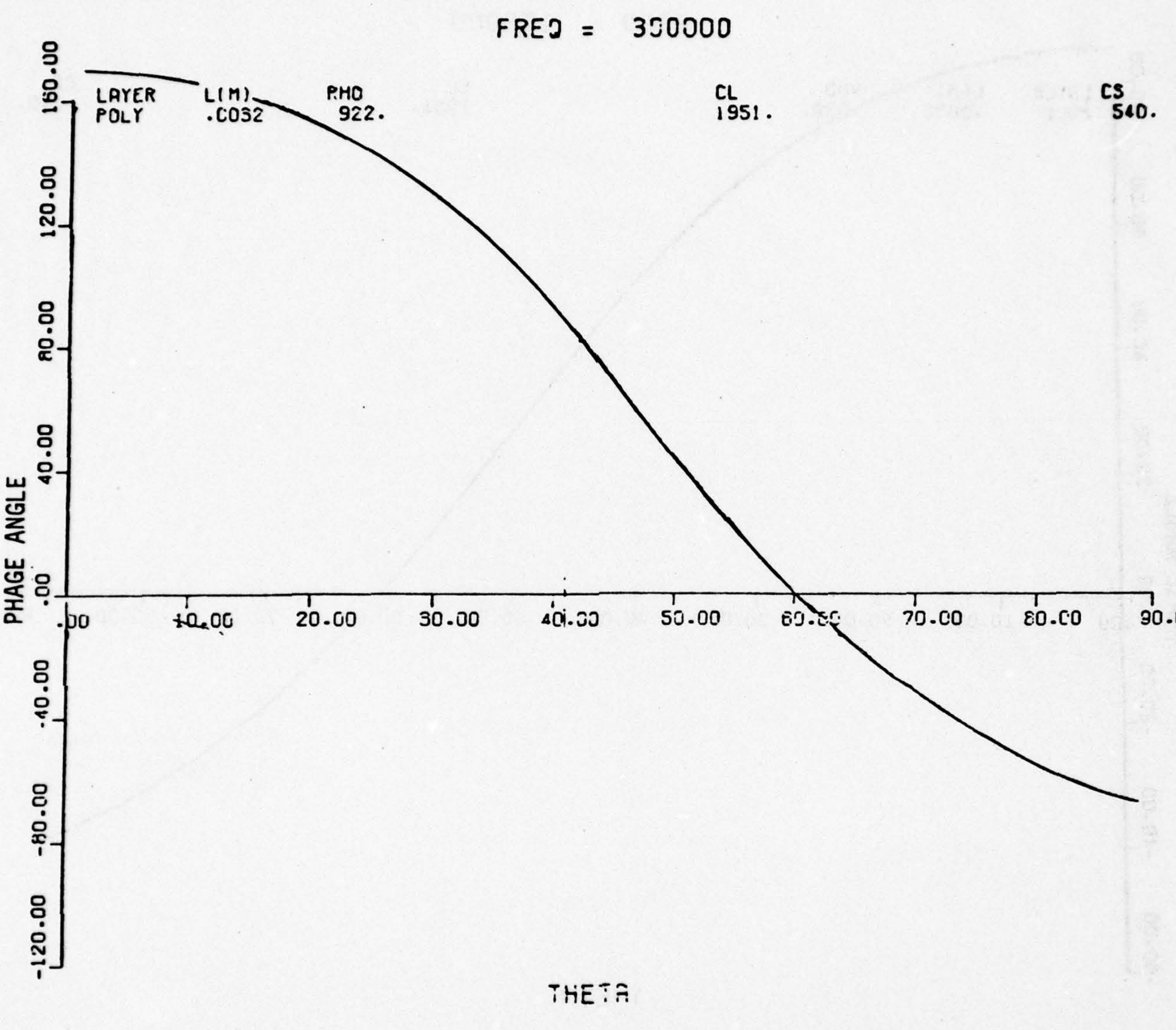


Figure 8(e). Phase Angle of Transmitted Wave vs Theta for Water/Polyethylene/FC-75

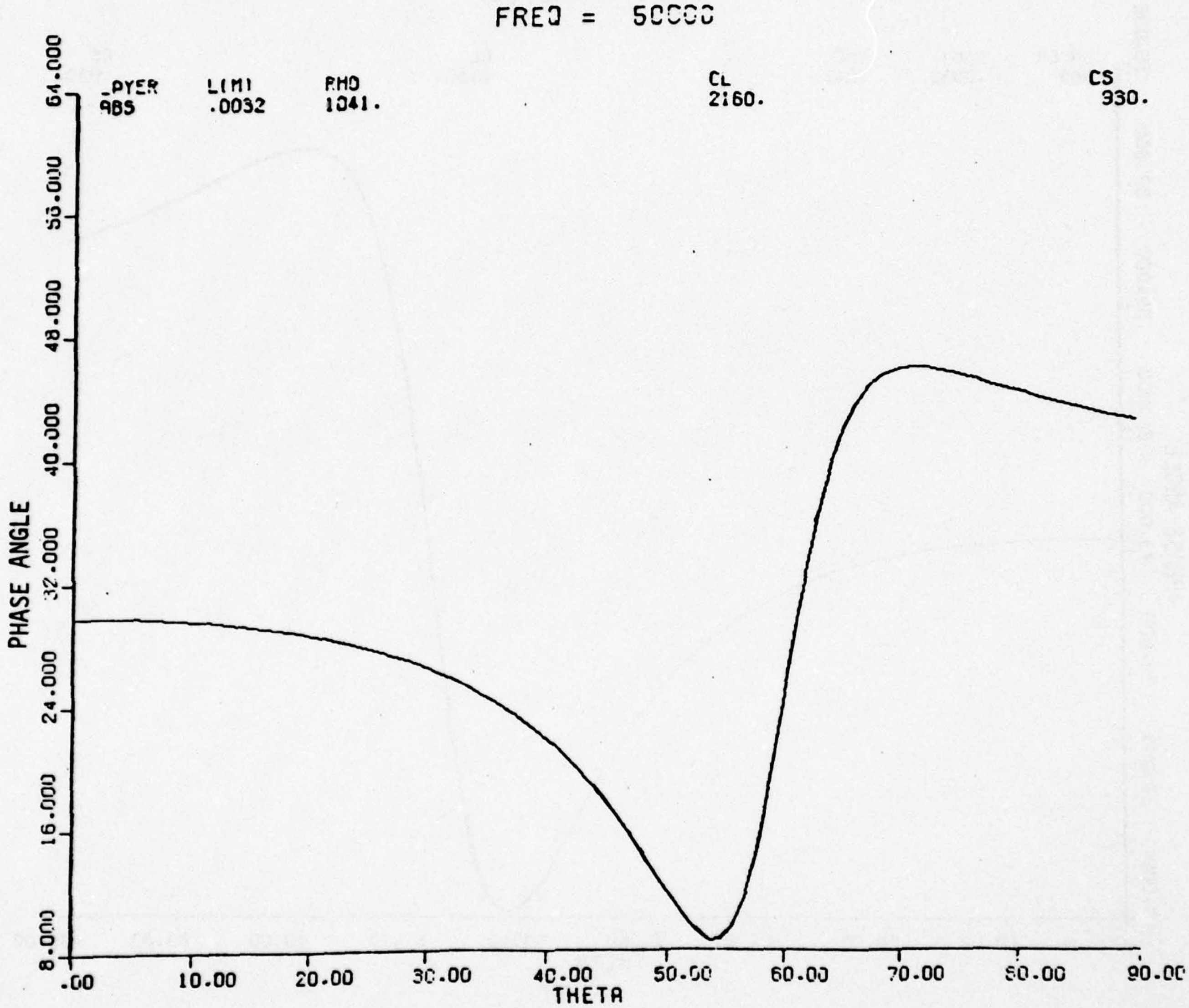


Figure 9(a). Phase Angle of Transmitted Wave vs Theta for Water/ABS/FC-75

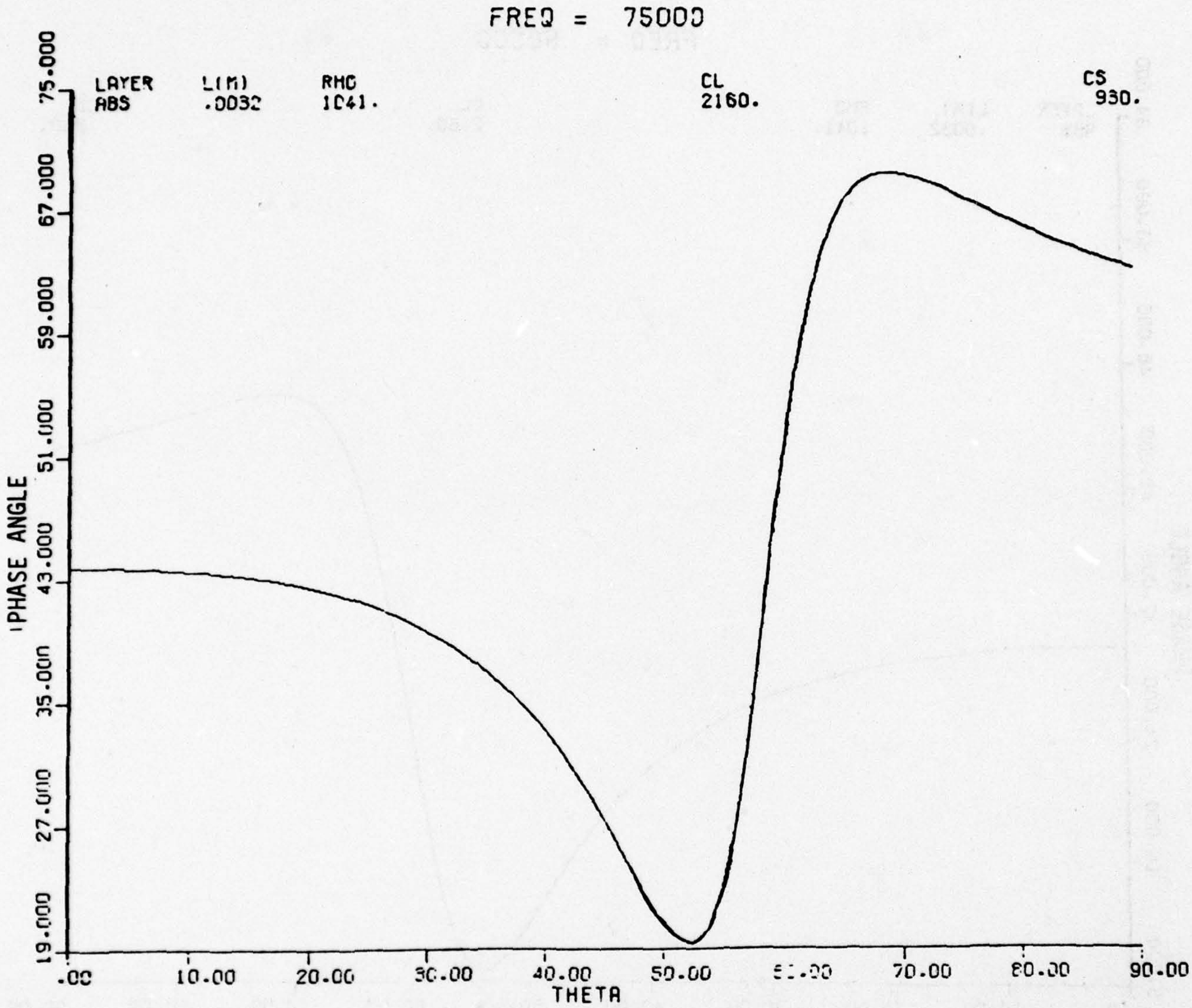


Figure 9(b). Phase Angle of Transmitted Wave vs Theta for Water/ABS/FC-75

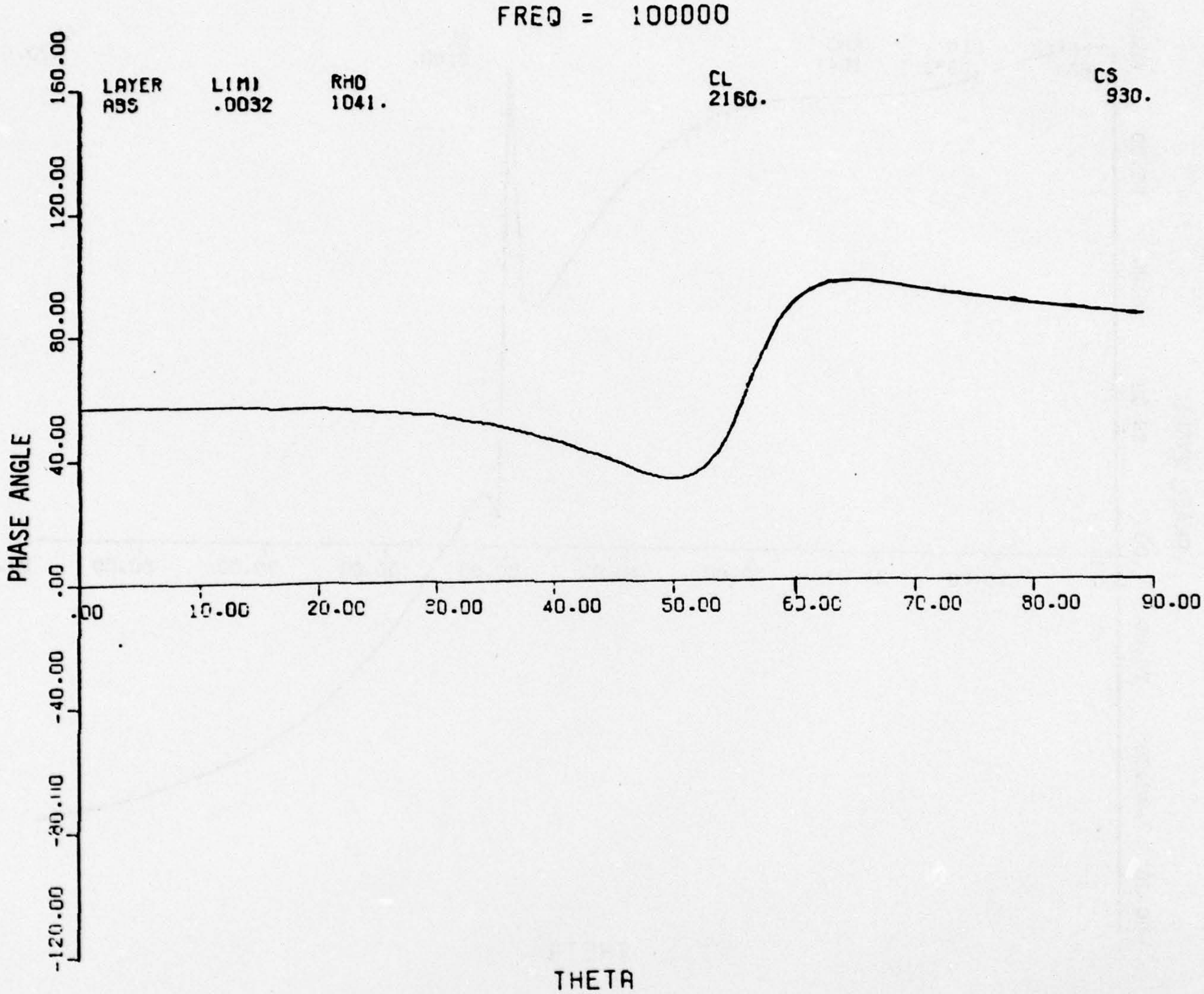


Figure 9(c). Phase Angle of Transmitted Wave vs Theta for Water/ABS/FC-75

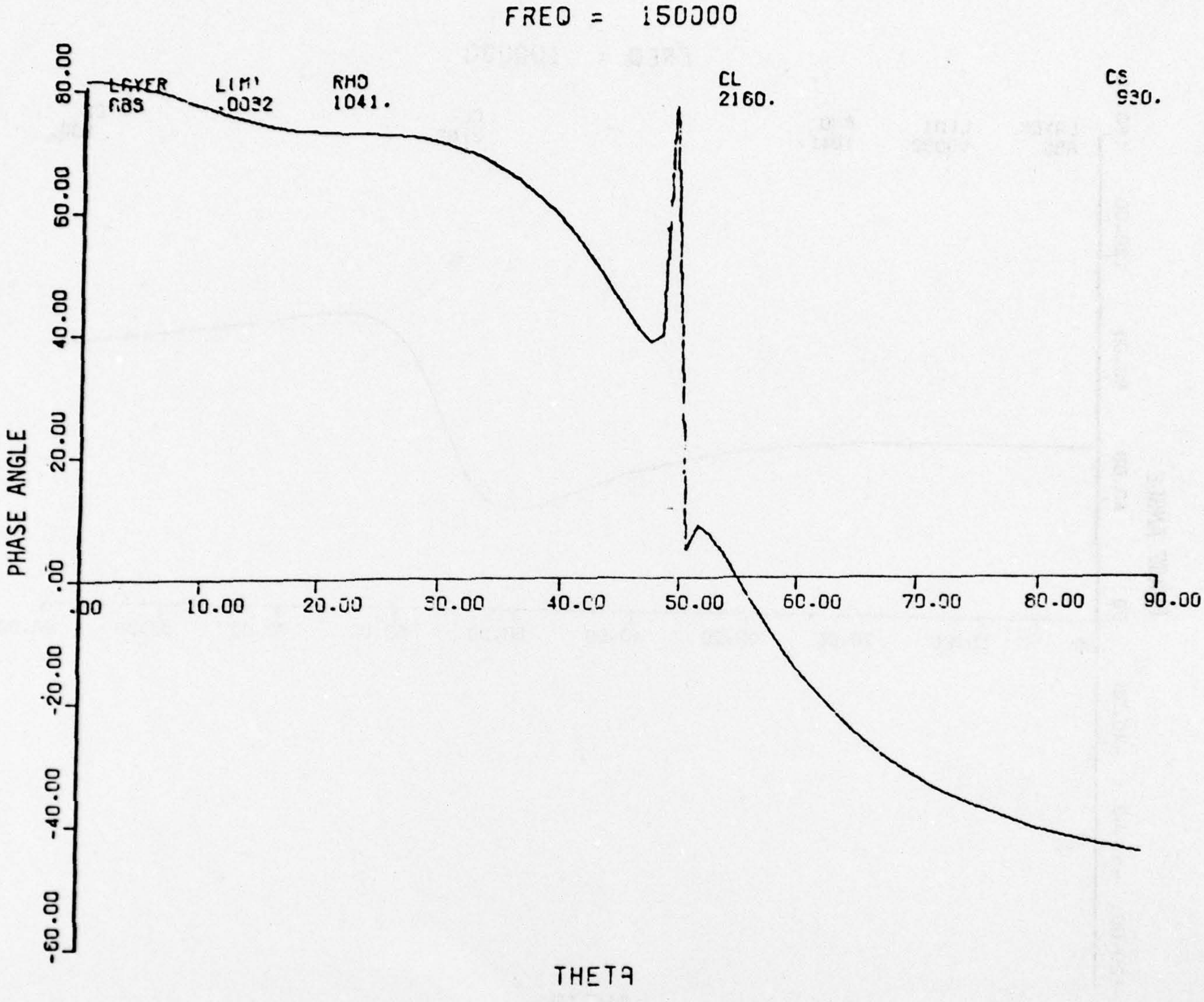


Figure 9(d). Phase Angle of Transmitted Wave vs Theta for Water/ABS/FC-75

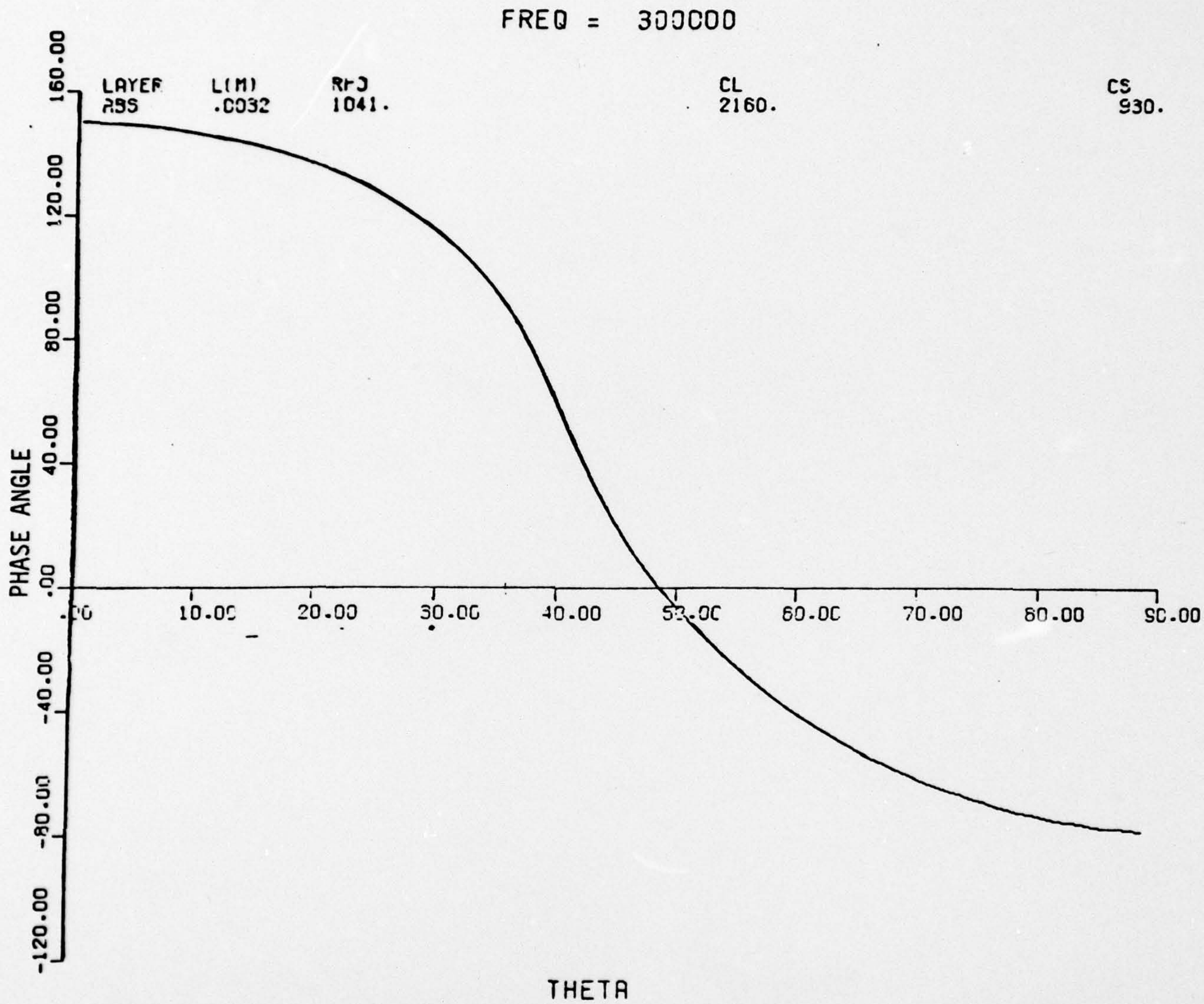


Figure 9(e). Phase Angle of Transmitted Wave vs Theta for Water/ABS/FC-75

# Section 4

# CONCLUSIONS

The data generated by the described method for computing transmission and reflection losses in multilayered media do show the coincidence rule to be effective in explaining the observed extrema. The cases where maxima are observed when minima are expected can be explained in terms of interaction between closely spaced modes.

Although the ABS exhibits a greater transmission loss than polyethylene over the whole spectrum because of its higher absorption, at low and moderate frequencies they both have good low loss characteristics and the sonar designer might select ABS because of its better structural properties. At higher frequencies the situation is different and the designer must be aware of the sensitivity of the transmission characteristics to the frequency, thickness of material and angle of incidence, factors which are somewhat controllable. By careful selection of frequency and thickness of material, "windows" of low transmission loss can be obtained. Care should be taken to avoid regimes where higher angles of incidence produce extreme peaks in the transmission loss curves.

Also of interest to the sonar designer is the change in phase of the transmitted wave as a function of the angle of incidence, particularly where beam forming is a consideration. The fluctuation in phase angle exhibited by ABS would make it a less satisfactory selection for a window material than the polyethylene which shows a smooth variation in phase over the range of incidence angles.

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(7) Measured Data on 3M Syntactic Foam, NSWC, White Oak, Maryland

(8) Brekhovskikh, L. M., Waves in Layered Media, (Academic Press, New York, 1960)

## Appendix A
## TRANSFER MATRIX ELEMENTS

For a single solid layer, the elements of the transfer matrix [M] are:

$$M_{11} = \frac{2K^2}{\overset{*}{k}_S^{\,2}} \cos\overset{*}{\xi} + \left(1 - \frac{2K^2}{\overset{*}{k}_S^{\,2}}\right) \cos\overset{*}{\eta}$$

$$M_{12} = \frac{iK}{\rho\omega^2} \left(\cos\overset{*}{\eta} - \cos\overset{*}{\xi}\right)$$

$$M_{13} = \frac{iK}{\overset{*}{K}_Z} \left(\frac{2K^2}{\overset{*}{k}_S^{\,2}} - 1\right) \sin\overset{*}{\xi} + \frac{2iK\overset{*}{K}_S}{\overset{*}{k}_S^{\,2}} \sin\overset{*}{\eta}$$

$$M_{14} = \frac{-1}{\rho\omega^2} \left(\frac{K^2}{\overset{*}{K}_Z} \sin\overset{*}{\xi} + \overset{*}{K}_S \sin\overset{*}{\eta}\right)$$

---

$$M_{21} = 2i\rho\omega^2 \frac{K}{\overset{*}{k}_S^{\,2}} \left(\frac{2K^2}{\overset{*}{k}_S^{\,2}} - 1\right) \left(\cos\overset{*}{\eta} - \cos\overset{*}{\xi}\right)$$

$$M_{22} = \left(1 - \frac{2K^2}{\overset{*}{k}_S^{\,2}}\right) \cos\overset{*}{\xi} + \frac{2K^2}{\overset{*}{k}_S^{\,2}} \cos\overset{*}{\eta}$$

$$M_{23} = \frac{\rho\omega^2}{\overset{*}{K}_Z} \left(\frac{2K^2}{\overset{*}{k}_S^{\,2}} - 1\right)^2 \sin\overset{*}{\xi} + \frac{4K^2\overset{*}{K}_Z\overset{*}{K}_S}{\overset{*}{k}_S^{\,4}} \sin\overset{*}{\eta}$$

$$M_{24} = M_{13}$$

$$M_{31} = \frac{-iK}{\overset{*}{K}_S} \frac{2\overset{*}{K}_S\overset{*}{K}_Z}{\overset{*}{k}_S^{\,2}} \mathrm{SIN}\overset{*}{\xi} + \left(\frac{2K^2}{\overset{*}{k}_S^{\,2}} - 1\right) \mathrm{SIN}\overset{*}{\eta}$$

$$M_{32} = \frac{-1}{\rho\omega^2}\left(\overset{*}{K}_Z \ \mathrm{SIN}\overset{*}{\xi} + \frac{K^2}{\overset{*}{K}_S} \mathrm{SIN}\overset{*}{\eta}\right)$$

$$M_{33} = \left(1 - \frac{2K^2}{\overset{*}{k}_S^{\,2}}\right)\mathrm{COS}\overset{*}{\xi} + \frac{2K^2}{\overset{*}{k}_S^{\,2}} \mathrm{COS}\overset{*}{\eta}$$

$$M_{34} = M_{12}$$

---

$$M_{41} = \frac{\rho\omega^2}{\overset{*}{K}_S} \frac{4K^2\overset{*}{K}_Z\overset{*}{K}_S}{\overset{*}{k}_S^{\,4}} \mathrm{SIN}\overset{*}{\xi} + \left(\frac{2K^2}{\overset{*}{k}_S^{\,2}} - 1\right)^2 \mathrm{SIN}\overset{*}{\eta}$$

$$M_{42} = M_{31},\ M_{43} = M_{21},\ M_{44} = M_{11}$$

where:

$$K = k_I \ \mathrm{SIN}\theta_I = (\omega/C_I)\ \mathrm{SIN}\theta_I$$

$$\overset{*}{K}_Z = (\overset{*}{k}_L^{\,2} - K^2)^{1/2}$$

$$\overset{*}{K}_S = (\overset{*}{k}_S^{\,2} - K^2)^{1/2}$$

$$\overset{*}{k}_L = \omega/\overset{*}{C}_L$$

$$\overset{*}{k}_S = \omega/\overset{*}{C}_S$$

$$\overset{*}{\xi} = \overset{*}{K}_Z d$$

$$\overset{*}{\eta} = \overset{*}{K}_S d$$

$\theta_I$ = angle of incidence

$C_I$ = velocity in input fluid

$\overset{*}{C}_L$ = longitudinal velocity in solid layer

$\overset{*}{C}_S$ = shear velocity in solid layer

$\rho$ = density of solid layer

d = thickness of solid layer

Applying the fluid-solid boundary conditions to the resulting matrix equation, the reflection and transmission coefficients in terms of displacement potentials can be computed as follows:

$$R = \frac{X - Y}{X + Y} \text{ and } S = \frac{2}{X + Y}$$

where:

$$X = \frac{1}{\rho_I \omega^2}\left(\rho_F \omega^2 A + iK_Z^F B\right)$$

$$Y = \frac{1}{iK_Z^I}\left(\rho_F \omega^2 C + iK_Z^F D\right)$$

$$A = M_{22} - \frac{M_{21}M_{42}}{M_{41}}$$

$$B = \frac{M_{21}M_{43}}{M_{41}} - M_{23}$$

$$C = \frac{M_{31}M_{42}}{M_{41}} - M_{32}$$

$$D = M_{33} - \frac{M_{31}M_{43}}{M_{41}}$$

$$K_Z^I = \left(k_I^{\,2} - K^2\right)^{1/2} = k_I \cos\theta_I$$

$$K_Z^F = \left(k_F^{\,2} - K^2\right)^{1/2}$$

$$k_F = \omega/C_F$$

$C_F$ = velocity in final fluid

$\rho_I$ = density of initial fluid

$\rho_F$ = density of final fluid

The displacement potentials are (Figure A-1):

$$\phi_I = e^{i\vec{k}_I \cdot \vec{r}} \text{ for the incident wave,}$$

$$\phi_R = Re^{i\vec{k}_R \cdot \vec{r}} \text{ for the reflected wave,}$$

$$\phi_T = Se^{i\vec{k}_F \cdot \vec{r}} \text{ for the the transmitted wave,}$$

where in general, the displacement is

$$\xi = \nabla\phi,$$

and the amplitude of the acoustic pressure is given by,

$$|P| = \rho C\omega|\xi| = \rho\omega^2|\phi|.$$

Since $TL = 10 \log \frac{I_F}{I_I}$,

where $I_F$ and $I_I$ are the respective acoustic intensities,

and $I = \frac{|P|^2}{2\rho C}$,

then

$$TL = 10 \log \left[\frac{\rho_F\, C_I}{\rho_I\, C_F} |\phi_T|^2\right]$$

$$= 10 \log \left[\left(\frac{\rho_F}{\rho_I}\right)\left(\frac{C_I}{C_F}\right) |S|^2\right].$$

Similarly, $RL = 10 \log |R|^2$.

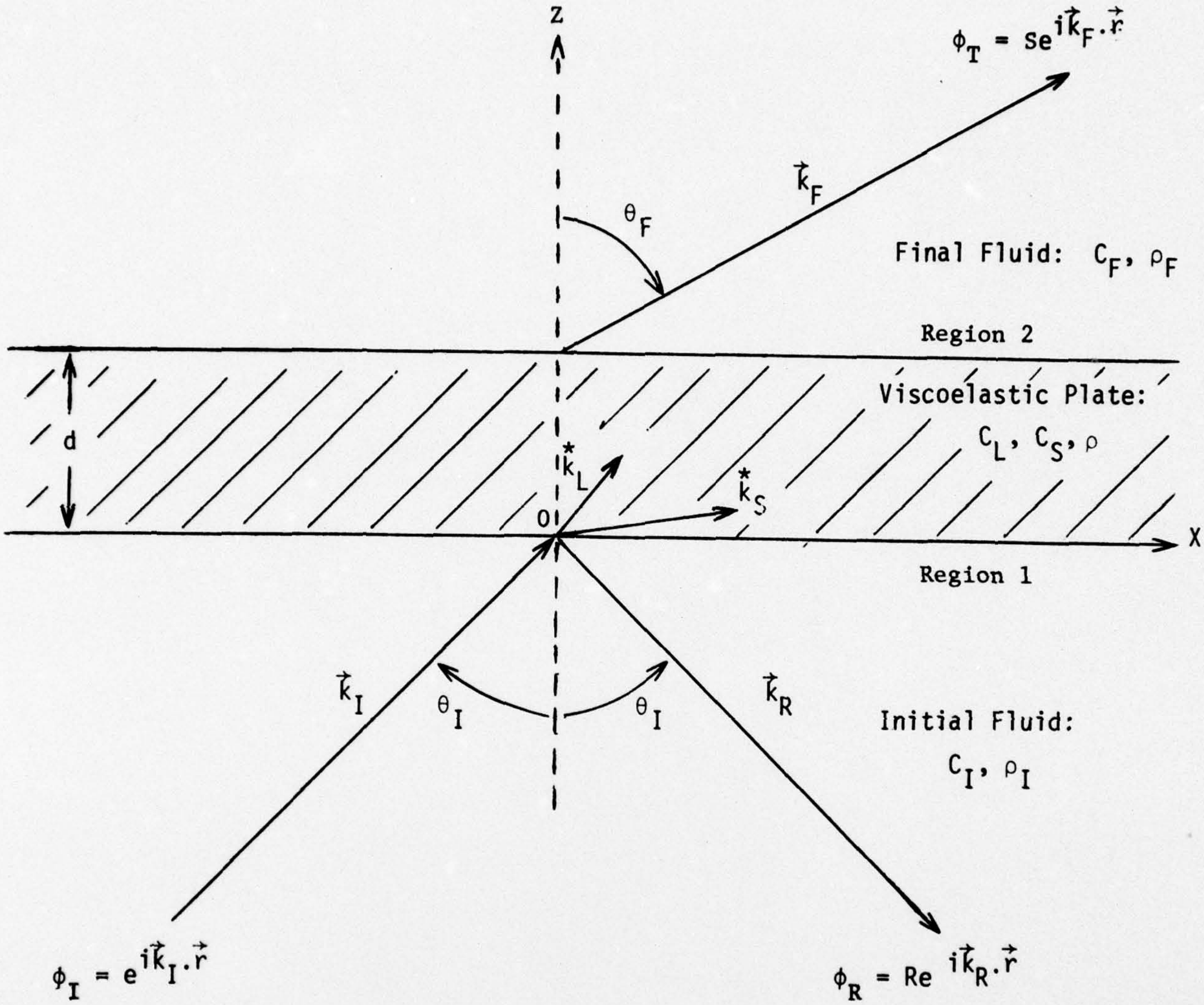


Figure A-1. Single Layer Problem

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